PHYSICS 208 Exam 3
Summer 2006

Do NOT open out the exam until instructed to do so!

Name: __________________________ UIN: __________________________

Signature: __________________________ E-mail: __________________________

Section Number: _____________ (5 points off for a wrong sec. #)

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<td>Derchyn Jong</td>
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<td>MW 2-4:40pm</td>
<td>Kevin Resil</td>
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<td>Jose Valadez</td>
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<td>304</td>
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<td>Derchyn Jong</td>
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<td>Jose Valadez</td>
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- You have the full class period to complete the exam.
- Formulae are provided on a separate colored sheet. You may NOT use any other formula sheet.
- When calculating numerical values, be sure to keep track of units.
- You may use this exam or come up front for scratch paper.
- Be sure to put a box around your final answers and clearly indicate your work to your grader.
- Clearly erase any unwanted marks. No credit will be given if we can’t figure out which answer you are choosing, or which answer you want us to consider.
- Partial credit can be given only if your work is clearly explained and labeled.
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Problem 1 (25 points): Invention!

You are inspired by a PHYS 208 lecture to design an electromagnetic rail gun that could accelerate payloads into earth orbit or beyond. You sketch the rail gun as below, where you model a conducting bar (mass $M$ and length $L$) as the payload. The bar slides over horizontal rails that are connected to a voltage source (not shown in the sketch). The voltage source maintains a constant current $I$ in the rails and bar. A constant, uniform, vertical magnetic field $\vec{B}$ fills the region between the rails. Before further design work, you first want to see a feasibility of the design by estimating the distance ($D$) the bar must travel along the rails if it is to reach the escape speed for the earth ($v_{\text{esc}} = 11.2$ km/s). You ignore the friction, air resistance, and electrical resistance for simplicity.

(a) (5 pts) Find the direction of the magnetic force on the bar.

(b) (15 pts) Express the distance $D$ in terms of $B (=|\vec{B}|)$, $I$, $M$, $L$, and/or $v_{\text{esc}}$.

(c) (5 pts) Let $B = 0.50$ T, $I = 2.0 \times 10^3$ A, $M = 25$ kg, and $L = 0.50$ m. Find the numerical value of $D$ and discuss your conclusion of the feasibility. (Namely, you will further work on the design? Explain why.)

Problem 1 (27.66) Solutions: Motion of a current-carrying wire in uniform magnetic field

a) Use $\vec{F} = i \vec{l} \times \vec{B}$. “to the right (or equivalent answer)”

b) $v^2 = 2aD \Rightarrow D = \frac{v^2}{2a} = \frac{v^2M}{2ILB}$. (use $a = F/M$ and $F = ILB$)

Mistake #1: This is treated as a motion with a constant velocity. This is a motion with a constant acceleration.

Mistake #2: $|\vec{B}| = \frac{\mu_0 I}{2\pi r} \Rightarrow D = \frac{\mu_0 I}{2\pi |\vec{B}|}$

c) $D = \frac{(1.12 \times 10^4 \text{ m/s})^2 (25 \text{ kg})}{2(2000 \text{ A})(0.50 \text{ m})(0.50 \text{ T})} = 3.14 \times 10^6 \text{ m} = 3140 \text{ km}$. (compared to $R_{\text{Earth}} = 6380$ km).

[Example 1] The original goal (to reach to the escape speed) is not possible. So yes I will give up the design.

[Example 2] The original goal (to reach to the escape speed) is not possible, but it would still be good for others as long as the final speed is required to be of order of 100 m/s or less, corresponding to $D$ being 300 m or less. I will think about any other purpose.
Problem 2 (25 points): \( I \) is the source of \( B \). Wrong grammar? No. It is an important physics topic.

(a) (10 pts) You learned the magnetic field \( \vec{B} \) produced by a straight current-carrying conductor of length \( 2a \) (see the figure).

The magnitude of \( \vec{B} \) is expresses as:

\[
|\vec{B}| = \frac{\mu_0 I}{4\pi} \int_a^{-a} \frac{x dy}{(x^2 + y^2)^{3/2}}
\]

Here you used \( \int \frac{x dy}{(x^2 + y^2)^{3/2}} = \frac{1}{x} \frac{y}{\sqrt{x^2 + y^2}} \). Derive the above equation using Biot-Savart’s law.

Problem 2 (Lecture) Solutions: How to calculate B-field using Biot-Savart Law

Review P.1071 as well as my lecture note. Note Eq. 2 is obtained when you take an infinitely long wire.
(b) (15 pts) You also learned a possible application of the above equations. For example, you can find the magnetic field $\mathbf{B}$ produced by a square current loop at any point of the $x$-$y$ plane. Find $\mathbf{B}$ (magnitude and direction) at point $(2L, 0)$.

Equation 2 of solutions in part (a) is mistakenly taken as the formula for this problem.
Problem 3 (15 points): This was an in-class quiz!
Express the magnitude of the magnetic flux in the loop, the magnitude of the rate change of the flux, and the current and its direction (c.w. or c.c.w.). The magnetic field is directed out of the page.

A rectangular wire loop, moving at a constant speed \( v \) in uniform magnetic field.

Problem 3 Solutions
This is to test a basic knowledge of Faraday’s law of induction. The concept is similar to Problems 9, 25, 49, 53, 61 of Ch. 29. None of them are the HW problems, but this problem is given as in-class quiz and solved together in the July 17 class, reviewed again on July 19, and posted on my PHYS-208 home page.
Problem 4 (25 points): Enjoy at PHYS-208 lab!
Your group carries out two experiments on a circuit containing an inductor \((L = 0.300 \text{ H})\), two resistors \((R_1 = 40.0 \Omega, R_2 = 25.0 \Omega)\), and an ideal battery with \(\varepsilon = 60.0 \text{ V}\). Switch S is opened initially.

(a) (15 pts) Switch S is closed. At some time \(t\) afterward the current in the inductor is increasing at a rate of \(\frac{di}{dt} = 50.0 \text{ A/s}\). At this instant, what is the current \(i_1\) through \(R_1\) and the current \(i_2\) through \(R_2\)?

(b) (10 pts) After the switch has been closed a long time, it is opened again. Just after it is opened, what is the current through \(R_1\)?

Problem 4 (30.70) Solutions: Understanding a time response of \(R-L\) circuit

a) Switch is closed, then at some later time:
\[
\frac{di}{dt} = +50.0 \text{ A/s} \Rightarrow V_a - V_c = -L \frac{di}{dt} = -(0.300 \text{ H}) (+50.0 \text{ A/s}) = -15.0 \text{ V}.
\]
The top circuit loop: \(60.0 \text{ V} - i_1 R_1 = 0 \Rightarrow i_1 = \frac{60.0 \text{ V}}{40.0 \Omega} = 1.50 \text{ A}.

The bottom loop: \(60 \text{ V} - i_2 R_2 - 15.0 \text{ V} = 0 \Rightarrow i_2 = \frac{45.0 \text{ V}}{25.0 \Omega} = 1.80 \text{ A}.

b) After a long time: \(i_2 = \frac{60.0 \text{ V}}{25.0 \Omega} = 2.40 \text{ A}\), and immediately when the switch is opened, the inductor maintains this current, so \(i_1 = i_2 = 2.40 \text{ A}\).
Problem 5 (10 points): Electric charge and magnetic pole equivalence

Below is what you have learned in the supplemental material about the “magnetic pole” equivalence.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Electricity</th>
<th>Magnetism</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge</td>
<td>( q ) (C)</td>
<td>( q_m ) (A-m)</td>
</tr>
<tr>
<td>Field</td>
<td>( \vec{E} ) (N/C=V/m)</td>
<td>( \vec{B} ) (T=N/A-m)</td>
</tr>
<tr>
<td>Force</td>
<td>( q\vec{E} ) (N)</td>
<td>( q_m\vec{B} ) (N)</td>
</tr>
<tr>
<td>Coupling Constant</td>
<td>( k \equiv \frac{1}{4\pi \epsilon_0} \approx 9.0 \times 10^9 \text{N-m}^2/C^2 )</td>
<td>( k_m \equiv \frac{\mu_0}{4\pi} = 1.0 \times 10^{-7} \text{N/A}^2 )</td>
</tr>
<tr>
<td>Point Source</td>
<td>( \vec{E} = kq \frac{\vec{r}}{r^2} )</td>
<td>( \vec{B} = k_m q_m \frac{\vec{r}}{r^2} )</td>
</tr>
<tr>
<td>Charge/Area</td>
<td>( \sigma \text{(C/m}^2)</td>
<td>( \sigma_m \text{(A/m)} )</td>
</tr>
<tr>
<td>Sheet Source</td>
<td>(</td>
<td>\vec{E}</td>
</tr>
<tr>
<td>Dipole Moment</td>
<td>( p = ql )</td>
<td>( \mu = q_m l )</td>
</tr>
<tr>
<td>Dipole Moment/Volume</td>
<td>Polarization ( P = p/V )</td>
<td>Magnetization ( M = \mu/V )</td>
</tr>
</tbody>
</table>
| Torque on Dipole in Field     | \( \vec{p} \times \vec{B} \) | -
| Energy of Dipole in Field     | \( -\vec{p} \cdot \vec{B} \) | -

Now you have two identical bar magnets of length \( l = 30 \text{ cm} \) and 0.3 cm-by-0.5 cm cross-sections in a PHYS-208 lab. Each has magnetic dipole moment 0.28 \text{ A-m}^2. They are placed in a line as in the figure. The distance between two bar magnets is \( r = 1.2 \text{ cm} \). Find their force (direction and magnitude).

Problem 5 Solutions

\[ \vec{F} \text{ Left bar} = \vec{F}_0 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 \]

\[ \vec{F}_0 = k_m q_m^2 \left[ \frac{1}{(r+l)^2} \hat{i} + \frac{1}{r^2} \hat{r} \left( -\hat{i} \right) \right] \]

\[ \vec{F}_2 = k_m q_m^2 \left[ \frac{1}{(r+2l)^2} \hat{i} + \frac{1}{(r+l)^2} \hat{i} \right] \]

\[ \vec{F}_3 = -k_m q_m^2 \frac{2}{(r+l)^2} \hat{i} \]

\[ \vec{F}_4 = -6.93 \times 10^{-3} \text{ [m}^2\text{]} \hat{r} \]

\[ \vec{F}_{\text{Left bar}} \approx \vec{F}_0 \approx -6.05 \times 10^{-4} \text{ [N]} \hat{i} \]

Magnitude = 6.04 \times 10^{-4} \text{ [N]}

Direction = “to left” on the left bar

Repulsive force between 2 bar magnets