ATM
PHYSICS 208 EXAM FINAL-B: SPRING 1995
(120 minutes)

Name: 

Section Number:

Student Number:

Contents Covered: All chapters.

Scores: 200 points

Notes:

- Don’t forget to specify the units.
1. **(25 points)** A long straight nonconducting rod of radius 1 cm has a uniform charge density $6 \times 10^{-3} \text{ C/m}^3$. Note $\varepsilon_0 = 8.8542 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$.

(a) Calculate the electric flux $\phi_E$ through a coaxial cylindrical surface (Gaussian surface) of radius 10 cm and length 100 cm.

(b) Find the magnitude of the electric field $\mathbf{E}$ at $r = 10$ cm.

(c) Find the electric potential difference $|\Delta V|$ between $r = 10$ cm and $r = 1$ cm.

2. **(25 points)** A long cylindrical conductor ($\rho = 1.7 \times 10^{-8} \Omega\cdot\text{m}$) of radius $R$ carries a current $I$. The current density $J$ is uniform over the cross section of the conductor. Assume $R = 1$ cm, $I = 1$ A. Note $\mu_0 = 4\pi\times10^{-7}$ Wb/A$\cdot$m.

(a) Find the current density $J$.

(b) Find the magnitude of the magnetic field $\mathbf{B}$ at a distance $r = 2R$ measured from the axis.

(c) Find the magnitude of the magnetic field $\mathbf{B}$ at a distance $r = \frac{1}{2}R$.

(d) Find the magnitude of the electric field $\mathbf{E}$ inside the conductor.

(e) Find the direction of the induced current in the loop as shown in the figure when the current is rapidly decreasing.

(f) Find the magnetic force on the electron ($q = 1.6\times10^{-19}$ C) whose speed is 10 m/s to the right at $r = 10$ cm.

3. **(25 points)** A capacitor is constructed from two square metal plates of side length $L$ and separated by a distance $d$. One half of the space between the plates is filled with polystyrene ($\kappa = 2.56$), and other half is filled with neoprene rubber ($\kappa = 6.7$). Note $\varepsilon_0 = 8.8542 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$. Find the capacitance of the device taking $L = 2$ cm and $d = 0.50$ mm.

4. **(25 points)** A series $RLC$ AC circuit is designed to have a quality factor $Q_0 = 100$ at $\omega_0 = 10$ Mrads/sec with $v = V_m \sin \omega t$, where $V_m = 100$ V. Take $L = 20 \mu\text{H}$.

(a) Find the capacitance $C$ and the resistance $R$.

(b) Find the average power consumption in the circuit at $\omega = \omega_0$.

(c) Find the amplitude of the voltage across the inductor at $\omega = \omega_0$. 
(d) Find the minimum and maximum angular frequencies of the AC generator at which the power consumptions are half its maximum value obtained in (b).

5. (25 points) Suppose that the switch has been closed sufficiently long for the capacitor to become fully charged.

(a) Find the steady-state current through each resistor and the charge $Q$ on the capacitor.

(b) The switch is now opened at $t = 0$. Find the initial current in the circuit. Also find the time that it takes for the charge on the capacitor to fall to 1/5 of its initial value.

6. (25 points) A plane electromagnetic wave has an energy flux of 750 W/m$^2$. A flat, rectangular surface of dimensions 50 cm $\times$ 100 cm is placed perpendicular to the direction of the plane wave. The surface absorbs half of the energy and reflect half (that is, it is a 50% reflecting surface). Take the speed of light in air to be $c = 3 \times 10^8$ m/s.

(a) Is the electromagnetic wave a transverse wave or a longitudinal wave? Also give one example for each wave.

(b) Find the momentum of the incident electromagnetic wave in a time of 1 min.

(c) Calculate the total energy absorbed by the surface in a time of 1 min.

(d) Find the momentum absorbed by the surface in a time of 1 min.

7. (25 points) A light source is located in air ($n = 1.0$). An observer in water ($n = 1.33$) sees the light of wave length 600 nm at an apparent angle of 45° from the vertical through a layer of glass ($n = 1.52$) which is between air and water.

(a) Find the angle of the light source from the horizontal.

(b) What is the wave length of the light in air.

8. (25 points) An air wedge is formed between two glass plates ($n = 1.5$) in contact along one edge and slightly separated at the opposite edge. When illuminated with monochromatic light from above, the reflected light reveals a total of 85 fringes. Calculate the number of dark fringes that would appear if liquid ($n = 1.8$) were to replace the air between the plates.

Note: Include the dark band at zero thickness along the edge of contact between the two plates.
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(a) The total flux is estimated to be

\[ \Phi_E = \frac{Q}{\varepsilon_0} \text{(Gauss' Law)} \]

where

\[ Q = \frac{\pi R^2}{2 \pi R L} \]

\[ \phi_E = \frac{\pi R^2}{2 \pi R L} P = \frac{Q \pi (10^{-2})^2}{\varepsilon_0 8.8542 \times 10^{-12}} \]

\[ = \frac{4.26 \times 10^5}{2.13} \text{[N m}^2\text{]} \]

(b) \[ \Phi_E = \int E \cdot \hat{n} \, dA + \int E \cdot \hat{n} \, dA + \int E \cdot \hat{n} \, dA \]

\[ \Phi_E = \frac{R^2 \rho}{2 \varepsilon_0} \]

\[ |E| = \frac{R^2 \rho}{2 \varepsilon_0} \]

\[ = \frac{(10^{-2})^2 \times 6 	imes 10^{-3}}{2 \varepsilon_0 8.8542 \times 10^{-12}} \times \frac{1}{10^{-1}} \]
(1) cont'd

because \( \Phi_E = 0 \) \( \implies E = 0 \) \( \forall \mathbf{r} \)

\[ E(r) = \frac{RP}{\varepsilon_0} \frac{1}{r} \quad (r \geq R) \]

\[ \Delta V(r) = V(r_2) - V(r_1) \]

\[ = \int_{r_1}^{r_2} \frac{E(r) \, dr}{\varepsilon_0} = \frac{RP}{\varepsilon_0} \ln \left( \frac{r_2}{r_1} \right) \]

\[ = \frac{RP}{\varepsilon_0} \ln \frac{r}{R} \]

\[ \Delta V = \frac{10^{-2} \times 6 \times 10^{-3}}{8.85 \times 2 \times 10^{-12}} \ln \left( \frac{10^{-1}}{10^{-2}} \right) \]

\[ \approx 1.56 \times 10^7 \text{ [V/C] or [Volt]} \]
(2) 25 pts

(a) \[ N = \frac{Q}{e} = \frac{1 \times (5 \times 1.0)}{1.6 \times 10^{-19}} = 1.875 \times 10^{21} \text{ electrons} \]

\[ J = \frac{I}{A} \rightarrow A = \pi R^2 = \pi \times (10^{-2})^2 \]

\[ J = \frac{1}{\pi \times 10^{-4}} = 3.183 \times 10^3 \left[ \frac{A}{m^2} \right] \]

(b) Ampere's law: \( \oint \vec{B} \cdot d\vec{l} = \mu_0 I \)

\[ |\vec{B}| \cdot (2\pi R) = \mu_0 I; \gamma = 2R \]

\[ |\vec{B}| = \frac{\mu_0 I}{2\pi R} = \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 2 \times 10^{-2}} = 10^{-5} \text{ [T]} \]

(c) \[ |\vec{B}| \cdot (2\pi R) = \mu_0 \times J \times (\pi R^2) \]

\[ |\vec{B}| = \frac{\mu_0 J \pi R^2}{2\pi R} = \frac{4\pi \times 10^{-7} \times 3.183 \times 10^3}{2} \]

\[ = 2.6 \times 10^{-5} \text{ [T]} \]

\[ = 1.0 \times 10^{-5} \text{ [T]} \]
(d) \( J = \sigma E = \frac{1}{\rho} E \)

(i) \( E = \rho J = 1.7 \times 10^{-8} \times 3.183 \times 10^{-3} \)

= \( 5.41 \times 10^{-5} [N/C] \)

(e) No current

\( |\vec{F}| = \left| \frac{1}{2} \vec{v} \times \vec{B} \right| = \frac{1}{2} \vec{v} \cdot \vec{B} \)

\( = 1.6 \times 10^{-19} \times 10 \times \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 10^{-1}} \)

= \( 3.2 \times 10^{-24} [N] \)

(f) \( I \)
\[ C = k \varepsilon_0 \frac{A}{d} \]

\[ L = 2 \text{ cm} \]

\[ K_1 = 2.56 \quad K_2 = 6.7 \]

\[ d = 0.5 \text{ mm} \]

\[ C_1 = K_1 \varepsilon_0 \frac{L \times \frac{1}{2} L}{d} = K_1 \varepsilon_0 \frac{L^2}{2d} \]

\[ C_2 = K_2 \varepsilon_0 \frac{L \times \frac{1}{2} L}{d} = K_2 \varepsilon_0 \frac{L^2}{2d} \]

\[ C = C_1 + C_2 = (K_1 + K_2) \varepsilon_0 \frac{L^2}{2d} \]

\[ = (2.56 + 6.7) \times 8.85 \times 10^{-12} \times \frac{2 \times 10^{-2}}{10^{-3}} \]

\[ = 8.64 \times 3.28 \times 10^{-11} [F] \]
\( \Omega_0 = \omega_0 L \quad \rightarrow \quad R = \frac{\omega_0 L}{\Omega_0} \)

\( C = \frac{1}{L \omega_0^2} \)

\( = \frac{1}{20 \times 10^{-6} \times (10^7)^2} \)

\( = 5 \times 10^{-10} \quad [F] \)

\( P_{ave} = \frac{V_{rms}^2}{R} \quad (at \quad \Omega = \Omega_0) \)

\( P_{ave} = \frac{\left( \frac{V_m}{\sqrt{2}} \right)^2}{R} \)

\( = \frac{10^7 \times 20 \times 10^{-6}}{100} \)

\( = 2 \times 10^{10} \quad [W] \)

\( V_L = I_m X_L \quad \rightarrow \quad I_m = \frac{V_m}{R} \quad at \quad \Omega = \Omega_0 \)

\( X_L = \omega L \)

\( V_L = \frac{V_m \omega L}{R} = 10^4 \quad [V] \)
(d) \( Q_0 = \frac{\Delta \omega}{\Delta \omega} \rightarrow \Delta \omega = \frac{\omega_0}{Q} = \frac{10^7}{100} = 10^5 \)

\[
\begin{align*}
\left\{ \begin{array}{l}
\omega_{\text{min}} = 9.95 \text{ Mrad/s} \\
\omega_{\text{max}} = 10.05 \text{ Mrad/s}
\end{array} \right.
\end{align*}
\]

5 pts
(a) After steady-state conditions have been reached, there is no DC current through the capacitor. Thus:

For \( R_3 \):

\[
I_{R_3} = 0 \quad \text{(steady-state)}
\]

For the other two resistors, the steady-state current is simply determined by the 9-V emf across the 12-k\( \Omega \) and 15-k\( \Omega \) resistors in series:

For \( R_1 \) and \( R_2 \):

\[
I_{(R_1+R_2)} = \frac{E}{R_1 + R_2} = \frac{9 \text{ V}}{12 \text{ k}\Omega + 15 \text{ k}\Omega} = \frac{0.333 \text{ mA}}{(\text{steady-state})}
\]

(b) After the transient currents have ceased, the voltage across \( C \) is the same as the voltage across \( R_2 = IR_2 \) because there is no voltage drop across \( R_3 \). Therefore, the charge \( Q \) on \( C \) is

\[
Q = CV_{R_2} = C(IR_2) = (10 \mu\text{F})(0.333 \text{ mA})(15 \text{ k}\Omega) = 50.0 \mu\text{C}
\]

(c) When the switch is opened, the branch containing \( R_1 \) is no longer part of the circuit. The capacitor discharges through \((R_2 + R_3)\) with a time constant of \((R_2 + R_3)C = (15 \text{ k}\Omega + 3 \text{ k}\Omega)(10 \mu\text{F}) = 0.180 \text{ s}\). The initial current \( I_0 \) in this discharge circuit is determined by the initial voltage across the capacitor applied to \((R_2 + R_3)\) in series:

\[
I_0 = \frac{V_C}{R_2 + R_3} = \frac{IR_2}{(R_2 + R_3)} = \frac{(0.333 \text{ mA})(15 \text{ k}\Omega)}{(15 \text{ k}\Omega + 3 \text{ k}\Omega)} = 0.278 \text{ mA}
\]

Thus, when the switch is opened, the current through \( R_2 \) changes instantaneously from 0.333 mA (downward) to 0.278 mA (downward) as shown in Figure 28.87.

Thereafter, it decays according to

\[
t_{R_2} = I_0 e^{-t/(R_2+R_3)C} = (0.278 \text{ mA})e^{-t/(0.180 \text{ s})} \quad \text{(for } t > 0\text{)}
\]

(d) The charge \( q \) on the capacitor decays from \( Q_0 \) to \( Q_0/5 \) according to

\[
Q_0 = Q_0e^{-t/(R_2+R_3)C}
\]

\[
\frac{Q_0}{5} = Q_0e^{-t/(0.180 \text{ s})}
\]

\[
5 = e^{t/(0.180 \text{ s})}
\]

\[
\ln 5 = \frac{t}{0.180 \text{ s}} \quad \text{or} \quad t = (0.180 \text{ s})\ln 5 = 0.290 \text{ s}
\]

\[
0.290 \text{ sec} \quad \text{OK}
\]
25 pts

(a) Transverse wave

\[ \sqrt{\frac{P}{c}} = \frac{U}{c} = \frac{IAt}{c} \]

\[ = \frac{750 \times 0.5 \times 1 \times 60}{3 \times 10^8} = \frac{2.5 \times 10^{-5} \text{[J/s]}}{m} \]

(b) 50%\[ \frac{1}{2} IA \times t = \frac{750 \times 0.5 \times 1 \times 60}{2} = 1.13 \times 10^4 \text{[J]} \]

(c) \[ u_1, p_1 \]

\[ u_2, p_2 (\leq \frac{1}{2} p_1) \]

\[ p_1 = -\frac{1}{2} p_1 + p_{\text{block}} \]

(6) \[ p_{\text{block}} = \frac{3}{2} p_1 \]

\[ = 1.13 \times 10^{-4} \text{[J/s]}/m \]

\[ 6 \]

before

After

6
(7) \[ \lambda_3 = 600 \text{ nm} \leq 45^\circ \]

\[ \theta_2 \]

\[ \frac{\sin \theta_1}{n_2} = \frac{\sin \theta_2}{n_3} \]

Snell's law:

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

\[ n_2 \sin \theta_2 = n_3 \sin \theta_3 \]

(1) \[ n, \sin \theta_1 = n_3 \sin \theta_3 \]

\[(n_3 = 1.33) \]

\[ \sin \theta_1 = 1.33 \times \sin 45^\circ \]

\[ = 0.9405 \]

(2) \[ \theta_1 = 70.13^\circ \]

(3) \[ \theta = 90^\circ - 70.13^\circ = 19.87^\circ \]

(a) \[ \theta \]

(b) \[ \lambda_1 n_1 = \lambda_2 n_2 = \lambda_3 n_3 \]

\[ \lambda_1 = 600 \text{ nm} \times 1.33 = 798 \text{ nm} \]
Destructive interference:

\[ \delta = \left( m + \frac{1}{2} \right) \lambda_n + \frac{1}{2} \lambda_n \]

\[ = m \lambda_n \text{ } \leftarrow \text{ 10 pts} \]

\[ 2t = 84 \lambda_n = 84 \lambda_{\text{air}} \]

\[ t = \frac{42 \lambda_{\text{air}}}{2} \text{ } \leftarrow \text{ 5 pts} \]

\[ \delta = m \lambda_n \]

\[ m = x \sqrt{42 \times 1.8} \]

\[ = \frac{2x}{72} \text{ } \leftarrow \text{ 4 pts} \]

\[ = 144 \]

\[ \text{1st dark fringe} \]

\[ 5 \text{ pts} \]

\[ t = 42 \lambda_{\text{air}} \Rightarrow \frac{t}{\lambda_{\text{air}}} = 42 \]

\[ \Delta t = m \frac{\lambda_{\text{air}}}{n} \Rightarrow m = \frac{2t}{\lambda_{\text{air}}} = 2n \left( \frac{t}{\lambda_{\text{air}}} \right) \]