ATM

PHYSICS 219 EXAM II-A: SPRING 1992
(75 minutes)

Name: ___________________________ Section Number: ___________________________

Student Number: ___________________________

Contents Covered: Lorentz equation, Ampere’s law, Biot-Savart law, Faraday’s law, Lentz law, Gauss’s law

Scores: There are 5 problems which are chosen from textbook exercise or example from what has been taught in class, but not identical to the original ones.

Notes:
• Don’t forget to specify the units.
• Equations:

  \[ \vec{F} = q \vec{v} \times \vec{B} \]  (Lorentz equation)

  \[ \vec{F} = i \ell \times \vec{B} \]  (derived from Lorentz equation)

  \[ \oint \vec{B} \cdot d\vec{\ell} = \mu_0 i \]  (Ampere’s law)

  \[ d\vec{B} = \frac{\mu_0 i \ell \times \vec{r}}{4\pi r^3} \]  (Biot-Savart law)

  \[ \mathcal{E} = -\frac{d\Phi_B}{dt} \]  (Faraday’s law + Lentz law)

• If necessary, use the following:
  - \( \mu_0 = 4\pi \times 10^{-7} \) tesla-meter/ampere.
  - Properties of some particles:

<table>
<thead>
<tr>
<th>Particle</th>
<th>Mass [kg]</th>
<th>Charge [C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton</td>
<td>( 1.67 \times 10^{-27} )</td>
<td>( +1.6 \times 10^{-19} )</td>
</tr>
<tr>
<td>Electron</td>
<td>( 9.11 \times 10^{-31} )</td>
<td>( -1.6 \times 10^{-19} )</td>
</tr>
<tr>
<td>Neutron</td>
<td>( 1.67 \times 10^{-27} )</td>
<td>0</td>
</tr>
</tbody>
</table>
1. (30 points)

(A) Particles 1, 2, 3 follow the paths shown in figure as they pass through the magnetic field there. Identify the type (name) of each particle from proton, neutron, and electron.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Particle Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle 1</td>
<td>Proton</td>
</tr>
<tr>
<td>Particle 2</td>
<td>Neutron</td>
</tr>
<tr>
<td>Particle 3</td>
<td>Electron</td>
</tr>
</tbody>
</table>

Note: If all three are correct, then 10 points. Otherwise 0 points.

(B) Eight wires cut the page perpendicularly at the points shown in figure. A wire labeled with the integer $k$ ($k = 1, 2, \ldots, 6$) bears the current $k i_0$ where $i_0 = 2$ amperes. For those with odd $k$, the current flows down into the page, for those with even $k$ it flows up out of the page. Evaluate $\oint \mathbf{B} \cdot d\mathbf{l}$ along the closed path shown in the direction indicated by the arrowhead.

\[
\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (+i_0 + 3i_0 - 6i_0) \\
= -2\mu_0 i_0 \\
= -2 \times (4\pi \times 10^{-7}) \times 2 \\
= -5.03 \times 10^{-6} \text{[tesla meter]} 
\]

(C) Gauss's law for electricity is written as $\oint \mathbf{E} \cdot n dS = q/\varepsilon_0$. Similarly, for magnetism, it is written as $\oint \mathbf{B} \cdot n dS = 0$ whose right hand side is set to zero and is different from the equation for electricity. This is based on a conclusion forced by certain fact of magnetism. Explain the fact.

The fact is that isolated magnetic pole (monopole) is not yet discovered experimentally.
2. (20 points) A metal wire of mass \( m \) slides without friction on two horizontal rails spaced a distance \( d \) apart, as in figure. The track lies in a vertical uniform magnetic field \( B \). A constant current \( i \) flows from generator \( G \) along one rail, across the wire, and back down the other rail. Find the velocity (speed and direction) of the wire as a function of time, assuming it to be at rest at \( t = 0 \).

\[
\begin{align*}
\text{From } \vec{F} = i \vec{L} \times \vec{B}, \text{ the force on the sliding wire is} & \quad \vec{F} = i d B \vec{k} \\
\text{Now, } \vec{F} = m \frac{d \vec{v}}{dt} \ (\text{Newton's 2nd Law}) & \quad (\text{7}) \\
\text{so } m \frac{d \vec{v}}{dt} = i d B \vec{k}. & \quad (\text{7}) \\
\end{align*}
\]

\[
\begin{align*}
\vec{v} = \frac{i d B}{m} \int dt \vec{k} &= \left( \frac{i d B}{m} \cdot t + \vec{v}_0 \right) \vec{k} \\
&= \frac{i d B}{m} \vec{t} \\
\text{speed direction} & \quad (\text{6}) \\
\text{Note: } \vec{t} \text{ means a unit vector to the} & \quad
\end{align*}
\]
3. (20 points) Figure shows a long straight wire carrying current $i$. Show that the magnitude of the magnetic field $\vec{B}$ at point $P$, using Biot-Savart law, is given by

$$|\vec{B}| = \frac{\mu_0}{2\pi} \frac{i}{R}.$$ 

From Biot-Savart law:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i}{r^3} \frac{d\vec{l} \times \vec{r}}{r^2}. \quad (\text{x})$$

$$|d\vec{B}| = \frac{\mu_0 i}{4\pi} \frac{dx \sin \theta}{r^2}. \quad (\text{1})$$

Also

$$x = R \tan \phi \quad (\text{2})$$

$$R = r \cos \phi \quad (\text{3})$$

$$\theta = \phi + \frac{\pi}{2} \quad (\text{4})$$

From (2): $\frac{dx}{dy} = R \frac{d \left( \tan \phi \right)}{dy} = R \frac{1}{\cos^2 \phi} \quad (\text{1}) \Rightarrow dx = \frac{R}{\cos^2 \phi} \ dy \quad (\text{2})$.

From (3): $r = \frac{R}{\cos \phi} \quad (\text{3})$.

From (4): $\sin \theta = \sin \left( \phi + \frac{\pi}{2} \right) = \cos \phi \quad (\text{4})$.

Put (2'), (3') and (4') into (1):

$$|d\vec{B}| = \frac{\mu_0 i}{4\pi} \frac{R d\phi}{\cos^2 \phi} \left( \frac{\cos \phi}{R} \right)^2 \cos \phi = \frac{\mu_0 i}{4\pi} \frac{\cos \phi}{R} \ d\phi$$

$$\Rightarrow B = \int d\vec{B} = \frac{\mu_0 i}{4\pi} \frac{1}{R} \left[ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos \phi \ d\phi \right] = \frac{\mu_0 i}{4\pi R} \left[ \cos \phi \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{\mu_0 i}{2\pi R}$$

\[ (5) \]
4. **(20 points)** Two long wires a distance $d$ apart carry equal antiparallel currents $i$, as in figure. (a) Show that $|\vec{B}|$ at point $P$, which is equidistant from the wires, is given by

$$|\vec{B}| = \frac{2\mu_0 i d}{\pi (4R^2 + d^2)}.$$  

*(Hint: use the result of #3)*

(b) In what direction does $\vec{B}$ point.

\[\text{\textbf{Ans:}}\]

\[|\vec{B}_1| = |\vec{B}_2| = \frac{\mu_0 i}{2\pi l} \frac{l}{l},\]

where $l = \sqrt{(\frac{d}{2})^2 + R^2}$.

If the coordinates $x$ & $y$ are defined as shown in figure,

\[
\begin{align*}
|B_1y| &= |B_2y| \\ 
|B_x| &= |B_2x| \\
\end{align*}
\]

from symmetric configuration.

Therefore

\[
\begin{align*}
B_y &= 0 \\
B_x &= -|\vec{B}_1| \cos \theta - |\vec{B}_2| \cos \theta = -2 |\vec{B}_1| \cos \theta, \hspace{1cm} \text{\textbf{(1)}}
\end{align*}
\]

where $\vec{B} = \vec{B}_1 + \vec{B}_2$, $\cos \theta = \frac{d}{2l}$.

\[\text{\textbf{Ans:}}\]

\[|\vec{B}| = |B_x| = 2 \times \left( \frac{\mu_0 i}{2\pi} \frac{l}{l} \right) \times \frac{d}{2l} = \frac{\mu_0 i}{2\pi} \frac{d}{l^2}
\]

\[= \frac{\mu_0 i}{2\pi} \frac{d}{(\frac{d}{2})^2 + R^2} = \frac{\mu_0 i}{2\pi} \frac{4d}{d^2 + 4R^2}
\]

\[= \frac{2\mu_0 i d}{\pi (4R^2 + d^2)}.
\]

\[\text{\textbf{Ans:}}\]

\[
\text{From (1) (or/and Figure),}
\]

$-x$ direction (to the left; or equivalent words)
5. (20 points) In figure, the magnetic flux through the loop perpendicular to the plane of the coil and directed into the paper is varying according to the relation

\[ \Phi_B = 6 \sin^2 \frac{\pi}{4} t, \]

where \( \Phi_B \) is in webers and \( t \) is in seconds. (a) What are the magnitudes of the electromotive force \( (\mathcal{E}) \) induced in the loop when \( t = 1.0 \) & 3.0 sec? 
(b) What are the directions of the current through \( R \) at \( t = 1.0 \) & 3.0 sec?

(a) From \( \mathcal{E} = -\frac{d\Phi_B}{dt} \) (Faraday's law),

\[ |\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right| \]

\[ = 6 \times 2 \sin \frac{\pi}{4} t \times \frac{\pi}{4} \cos \frac{\pi}{4} t \]

\[ = 3 \pi \left| \sin \frac{\pi}{4} t \cos \frac{\pi}{4} t \right| \]

\[ t = 1 \quad 3 \pi \sin \frac{\pi}{4} \cos \frac{\pi}{4} = \frac{3}{2} \pi = \frac{4.71}{4.71} \]

\[ t = 3 \quad 3 \pi \sin \frac{3\pi}{4} \cos \frac{3\pi}{4} = \frac{3}{2} \pi = \frac{4.71}{4.71} \]

(b)

<table>
<thead>
<tr>
<th>( t )</th>
<th>Magnetic flux</th>
<th>Induced ( \mathbf{B} ) (Lenz's Law)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = 1 ) sec</td>
<td>( \frac{d\Phi_B}{dt} = 3\pi \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{3}{2} \pi &gt; 0 )</td>
<td>( \times ) increasing</td>
</tr>
<tr>
<td>( t = 3 ) sec</td>
<td>( \frac{d\Phi_B}{dt} = 3\pi \times \frac{1}{\sqrt{3}} \times (-\frac{1}{\sqrt{3}}) = -\frac{3}{2} \pi &lt; 0 )</td>
<td>( \times ) decreasing</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( t )</th>
<th>Direction of current</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = 1 ) sec</td>
<td>( \rightarrow )</td>
</tr>
<tr>
<td>( t = 3 ) sec</td>
<td>( \leftarrow )</td>
</tr>
</tbody>
</table>