Problem 1 (25 points): RLC circuit in ac source

Your PHYS202 lab team tested an RLC circuit with ac source (period $T_{\text{period}} = 0.628$ ms) and obtained the following beautiful voltage versus time graph (Curves 1-4) where the maximum current was $I_{\text{max}} = 0.1$ A. However, before submitting the lab report to your TA, you realized anyone in the team didn’t label Curves 1, 3, and 4, but Curve 2. Curve 2 shows $v_{\text{total}}$ where $v_{\text{total}} = v_R + v_L + v_C$.

(a) (12 pts) See the table below. Identify the correct labels ($v_R$, $v_L$, or $v_C$) for Curves 1, 3, and 4.

(b) (5 pts) Find the angular frequency ($\omega$).

(c) (3 pts) Find the numerical values (in ohm or $\Omega$) of the resistance ($R$), inductive reactance ($X_L$), and capacitive reactance ($X_C$).

(d) (2 pts) Find the numerical values of the inductance (in henry or $H$) and capacitance (farad or $F$).

(e) (3 pts) Construct the phase diagram at $t = T_{\text{period}}/4$.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$v_R$</th>
<th>$v_L$</th>
<th>$v_C$</th>
<th>$v_{\text{total}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 V</td>
<td>-60 V</td>
<td>60 V</td>
<td>0 V</td>
</tr>
<tr>
<td>0.157 ms</td>
<td>30 V</td>
<td>-40 V</td>
<td>20 V</td>
<td>20 V</td>
</tr>
</tbody>
</table>

(b) $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.628 \times 10^{-3}} = 10^4 \text{ rad/s}$

(c) $R = \frac{V_{\text{max}}}{I_{\text{max}}} = \frac{300}{0.1} = 3000 \Omega$

(d) $L = \frac{X_L}{\omega} = \frac{600}{10^4} = 60 \text{ mH}$

(e) $C = \frac{1}{\omega X_C} = \frac{1}{(10^4)(200)} = 5 \times 10^{-6} \text{ F}$
Version A

Problem 2 (25 points): EM Waves

You, as a member of Project "PHYS202 Exam3" team in National Aggie Spacecraft Agency (NASA), are working a conceptual design report of solar sailing. A solar sailing craft uses a large, low-mass sail and the energy and momentum of sunlight for propulsion. Your primary responsibility is to finalize the following specifications of the spacecraft:

(a) (10 pts) Should the material for sail be absorptive or reflective? Why?
(b) (15 pts) What is the minimum size of a sail to propel the spacecraft against the sun’s gravitational force \( F = G \frac{m M_{\text{sun}}}{r^2} \), where \( G = 6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \) and \( M_{\text{sun}} = 1.99 \times 10^{30} \text{ kg} \) if the spacecraft is placed at a distance of \( 1.5 \times 10^{11} \text{ m} \) from the sun? Express your result in square kilometers. Assume the mass of the spacecraft is 10,000 kg and the total power output of the sun is \( 3.9 \times 10^{26} \text{ W} \). Ignore the gravitation forces from all other planets.

\( \text{(a)} \) The material should be totally reflective, so that the radiation force is maximized.

\[
F_{\text{rad}} = \left( \frac{2 I}{c} \right) A
\]

\( \text{(b)} \)

\[
F_{\text{rad}} = F_{\text{grav}}
\]

\[
2 \frac{I}{c} \frac{A}{A} = G \frac{m M_{\text{sun}}}{r^2}
\]

\[
A = \frac{G m M_{\text{sun}}}{r^2}
\]

\[
\therefore A = 6.42 \times 10^6 \text{ m}^2 = 6.4 \text{ km}^2
\]

Correct concepts on (1) and (2) --- 10 pts
Calculation --- 5 pts
Version A

Problem 3 (25 points): Reflection and Refraction
You are on the top horizontal surface of a transparent solid block (index of refraction is \( n = 1.38 \)) and try to communicate with James Bond at the left side of the vertical surface \( A \) of the block by sending a laser beam signal. See the figure. However you know that there is a situation where the beam is totally reflected back into the block at the vertical surface, so that you are not able to contact him. You also know it depends on the angle \( \theta_a \) (measured from the horizontal surface). Calculate the range of \( \theta_a \) where you can successfully communicate with him.

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \text{(1)} \]
\[ \theta_1' \quad ? \quad 1.38 \quad ? \]
\[ n_2 \sin \phi_2 = n_3 \sin \theta_3 \quad \text{(2)} \]
\[ 1.38 \quad 1.0 \quad 90^\circ \]

\[ \text{(2)} \rightarrow \phi_2 = 46.438^\circ \quad \text{(3)} \quad \theta_2 = 90^\circ - \phi_2 = 43.561^\circ \]
\[ (\sin \theta_2 = 0.6891 \ldots) \]

\[ \text{(1)} \rightarrow \sin \theta_1 = (1.38)(0.6891 \ldots) = 0.950999 \ldots \]
\[ \therefore \theta_1 = 71.99^\circ = 72^\circ \]

Successful communication: \( 72^\circ < \theta_1 < 90^\circ \)

\[ 0^\circ < \theta_a < 18^\circ \]
Version A

Problem 4 (25 points): Geometric Optics

You examined an image formed by a concave mirror (see the figure below) in PHYS202 lecture in March 19. The concave mirror has a radius (distance between C and V) of curvature with 20 cm. You placed an object (I in the figure) perpendicular to the axis of the mirror at 16 cm from the vertex V.

(a) (5 pts) Calculate the position of the focal point (F) measured from V.

(b) (10 pts) Find graphically the image of the object by drawing at least two principal rays.

(c) (10 pts) Using the formula, calculate the image position (s') and lateral magnification (m) of the image. To receive a full credit the numerical results (including its sign) must be consistent with the graphical solution in part (b).

\[ FV = \text{(distance between F and V)} = \frac{\text{radius}}{2} = \frac{20 \text{ cm}}{2} = 10 \text{ cm} \]

(b) See above. (At least 2 principal rays)

(c) \[ \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \]

\[ \begin{cases} s = +16 \text{ cm} \\ s' = \text{unknown} \\ f = +10 \text{ cm} \end{cases} \]

\[ s' = +26.7 \text{ cm} \]

\[ m = -\frac{s'}{s} = -1.67 \]

"-" means that the image is inverted. \(|m| > 1 \Rightarrow\) The image is bigger.