Problem 1 (25 points):

(i) (5 pts) Consider the four field patterns shown. Assuming there are no charges in the regions shown, which of the patterns represent(s) a possible electric field:

1. (a)  
2. (b)  
3. (b) and (d)  
4. (a) and (c)  
5. (b) and (c)  
6. some other combination  
7. None of the above.

(ii) (12 pts) You prepare three metal spheres which are initially neutral as shown in (a) of each figure below. The figures show two examples of charging metal spheres by induction. Sketch the charge distribution on each sphere. Use “+” for positive charge and “−” for negative charge.

(iii) (8 pts) You saw this slide in my PHYS 202 class. A three-million-volt (3,000,000 V) lightning stroke hit the car containing a former Aggie who took PHYS 202 and passed at TAMU. He is calm instead of shocked. Explain why.

Fig. 12. An artificial, three-million-volt, lightning stroke does not hurt the passenger. (Courtesy of Aetlinghouse.)

Key: O Charges are on the surface.  
② [② of “+”]= [② of “−”]
Problem 4 (25 points): A small 1.5 g plastic ball is tied to a very light 2.00 cm string that is attached to the ceiling. See the figure. A uniform horizontal electric field exists between two plates. When the ball has been given an excess charge of \(-147 \ \mu C\), you observe that it remains suspended, with the string making an angle of 30.0° with the vertical. Find the magnitude and direction of electric field.

\begin{align*}
F &= 8 E \\
&\text{Since } q < 0, \text{ the direction of } \vec{E} \text{ is opposite to} \\
&\text{the direction of } \vec{F} \\
&\text{to the left.}
\end{align*}

\begin{align*}
\begin{cases}
F_T \cos \theta &= mg \quad \text{(1)} \\
F_T \sin \theta &= |q| E \quad \text{(2)} \\
|E| &= |E| \quad \text{(3)}
\end{cases}
\end{align*}

\begin{align*}
(1) &\Rightarrow F_T = \frac{mg}{\cos \theta} \quad \text{(3)} \\
(3) &\Rightarrow (2): \quad \frac{mg}{\cos \theta} \sin \theta = |q| E \\
\therefore E &= \frac{mg}{|q|} \tan \theta = \frac{(0.0015 \text{ kg}) (4.8 \text{ m/s}^2)}{147 \times 10^{-6} \text{C}} \\
&= 57.7 \text{ [N/C]}
\end{align*}

If this is correct, give a full credit.
Problem 3 (25 points): Your lab assignment is to characterize the circuit ($C_1 = C_2 = C_3 = 4.00 \ \mu F$ and $C_4 = 6.00 \ \mu F$) shown in the figure below. Before the measurements, your TA requests you to predict (i) the charge on each capacitor and (ii) the potential difference across each capacitor if the potential difference across $ab$ is +24 V. Also calculate (iii) the equivalent capacitance of the circuit.

\[ \frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \therefore \quad C_{12} = 2 \ \mu F \]

\[ C_{123} = C_{12} + C_3 \quad \therefore \quad C_{123} = 6 \ \mu F \]

\[ \frac{1}{C_{eq}} = \frac{1}{C_{123}} + \frac{1}{C_4} \quad \therefore \quad C_{eq} = 3 \ \mu F \]

\[ Q = C_{eq} V_{ab} = (3 \ \mu F)(24 \ V) = \boxed{72 \ \mu C} \quad \text{for } C_4 \]

\[ Q = C_{123} V_{123} \quad \therefore \quad V_{123} = \frac{72 \mu C}{6 \ \mu F} = \boxed{12 \ V} \quad \text{for } C_3 \]

\[ Q = C_4 V_4 \quad \therefore \quad V_4 = \frac{72 \mu C}{6 \ \mu F} = \boxed{12 \ V} \]

Check: $V_{123} + V_4 = 12 \ V = V_{ab}$

\[ Q_{12} = C_2 V_{12} \quad \therefore \quad Q_{12} = (2 \ \mu F)(12 \ V) = 24 \ \mu C \]

\[ Q_3 = C_3 V_{123} \quad \therefore \quad Q_3 = (4 \ \mu F)(12 \ V) = 48 \ \mu C \]

Check: $Q_{12} + Q_3 = 72 \ \mu C = Q$

\[ Q_{12} = C_1 V_1 \quad \therefore \quad V_1 = \frac{24 \mu C}{4 \ \mu F} = \boxed{6 \ V} \]

\[ Q_{12} = C_2 V_2 \quad \therefore \quad V_2 = \frac{24 \mu C}{4 \ \mu F} = \boxed{6 \ V} \]

Check: $V_1 + V_2 = 12 \ V = V_{12}$
Problem 2 (25 points): You design a parallel-plate air capacitor, which is made from two plates each having area \(4 \times 10^{-2} \text{ m}^2\) and being 0.800 cm apart. It is connected to a 120-V battery. (i) What is the capacitance? (ii) What is the charge on each plate? (iii) What is the electric field between the plates? (iv) What is the energy stored in the capacitor? (v) If the battery is disconnected and then the plates are pulled apart to a separation of 1.60 cm, what are the answers to parts (i), (ii), (iii), and (iv)?

\[
(i) \quad C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12} \text{ F/m}) (4 \times 10^{-2} \text{ m}^2)}{8 \times 10^{-3} \text{ m}} = 4.425 \times 10^{-11} \text{ F}
\]

\[
(ii) \quad Q = CV = (4.425 \times 10^{-11} \text{ F}) (120 \text{ V}) = 531 \times 10^{-11} \text{ C} = 5.31 \text{ nC}
\]

\[
(iii) \quad E = \frac{V}{d} = \frac{120 \text{ V}}{8 \times 10^{-3} \text{ m}} = \left[ 15 \times 10^3 \text{ V/m} \right]
\]

\[
(iv) \quad U = \frac{1}{2} CV^2 = \frac{1}{2} (4.425 \times 10^{-11} \text{ F}) (120 \text{ V})^2
\]

or \( \frac{Q^2}{2C} = 3.186 \times 10^{-7} \text{ J} = 3.186 \text{ mJ} \)

\[
(v) \quad \begin{cases}
C_{\text{new}} = \frac{\varepsilon_0 A}{2d} = \frac{1}{2} \times (4.425 \times 10^{-11} \text{ F}) = 2.21 \times 10^{-11} \\
Q_{\text{new}} = Q = 531 \times 10^{-11} \text{ C} \\
E_{\text{new}} = \frac{V_{\text{new}}}{2d} = \frac{Q_{\text{new}}/C_{\text{new}}}{2d} = \frac{Q \cdot 2/C}{2d} = \frac{Q/C}{d} = \frac{V}{d} \\
U_{\text{new}} = \frac{1}{2} C_{\text{new}} V_{\text{new}}^2 = \frac{1}{2} \left( \frac{1}{2} C \right) \left( \frac{Q}{C} \right)^2 = \frac{7}{16} \left( \frac{Q^2}{2C} \right)
\end{cases}
\]