Locally Supersymmetric Grand Unification

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A locally supersymmetric grand unification program is proposed which couples the $N=1$ supergravity multiplet to an arbitrary grand unified gauge group with any number of left-handed chiral multiplets and a gauge vector multiplet. A specific model is discussed where it is shown that not only do the gravitational interactions eliminate the degeneracy of the vacuum state encountered in global supersymmetry, but simultaneously they can break both supersymmetry and $SU(2) \otimes U(1)$ down to a residual $SU(5)^c \otimes U(1)$ symmetry at ~300 GeV.

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Recently much interest has been devoted to supersymmetric grand unified theories. All existing supersymmetric grand unified models are based on global supersymmetry. In such theories it is generally easy to break spontaneously the internal, e.g., $SU(5)$, symmetry, but more difficult to break supersymmetry itself. In this paper we propose a new type of supersymmetric grand unified model based on local supersymmetry. We consider here $N=1$ supergravity coupled to left-handed chiral scalar and gauge multiplets. We will see that the supergravity couplings automatically produce a spontaneous breaking which removes the degeneracy of the
different gauge vacua. We will consider two models in this paper. In a preliminary model the vacuum degeneracy is removed but supersymmetry and $\text{SU}(2) \otimes \text{U}(1)$ symmetry are maintained. In the second model, the supergravity interactions cause spontaneous breaking of both supersymmetry and $\text{SU}(2) \otimes \text{U}(1)$. Thus supersymmetry breaking occurs at the same scale as the $\text{SU}(2) \otimes \text{U}(1)$ breaking, i.e., $m_s \sim 300 \text{ GeV}$. The model is "realistic" in the sense that the mass hierarchy separating heavy and light particles can be maintained, and the low-energy phenomenology is correctly included. In addition, a variety of new physics is predicted at the teraelectronvolt range. As far as we know, this is the only model where electroweak spontaneous breaking is a consequence of gravitational interactions.

We begin by reviewing briefly the globally supersymmetric theories. For a given grand unified group $G$ one introduces a set of chiral left-handed $N = 1$ supermultiplets which form an arbitrary reducible representation of the group $G$. We shall denote these chiral supermultiplets collectively by $\Sigma^a = (\Sigma^a L, \Sigma^a, H^a)$ where

$$
\begin{align*}
\Sigma^a &= (A^a + i B^a, H^a = i F^a - i G^a), \\
\chi^a &= (1 - \gamma_5) x^a, \\
g_1 &= \lambda_1 \left( \frac{4}{3} \text{Tr} \Sigma^3 + \frac{1}{6} M \text{Tr} \Sigma^2 \right) + \lambda_2 H_x \Sigma^x + \lambda_3 M (\delta^x + 3 \delta^y \delta^x - 5 \delta^x \delta^y),
\end{align*}
(1)
$$

where $m_1$ and $m_2$ are matrices in the generation space.

The potential for Eq. (3) has a minimum when the vacuum expectation values for $H^x$, $H_x^x$, $H^y$, $H_x^y$, and $M_x^y$ vanish and when one of the following three solutions for the vacuum expectation value of $\Sigma$ holds:

(i) $\Sigma^x = 0$,

(ii) $\Sigma^x = \frac{1}{M} (\delta^x + 3 \delta^y \delta^x - 5 \delta^x \delta^y)$,

(iii) $\Sigma^x = \frac{1}{M} \delta^x + \delta^y \delta^x + \delta^y \delta^x$.

Solution (i) corresponds to no symmetry breaking, (ii) breaks $\text{SU}(5)$ into $\text{SU}(4) \otimes \text{U}(1)$, while (iii) breaks $\text{SU}(5)$ into $\text{SU}(3) \otimes \text{SU}(2) \otimes \text{U}(1)$. The three solutions are all degenerate and there is no way to lift the degeneracy within global supersymmetry. If one picks solution (iii), the masses

$$
L_b = \left( \frac{1}{\kappa^2} \right) \left( \mathcal{R} (\varphi, \omega) + \mathcal{R} (\kappa^2) \mathcal{G}_{\alpha \beta} \left[ \partial_{\mu} z^\alpha \partial^\beta + \frac{1}{4} i \mathcal{G}_{\alpha} (\partial^\mu z^\alpha) \right] \right),
$$

where $G = \kappa^2 / 8 \pi$ is the gravitational constant, $\mathcal{R}(\varphi, \omega)$ is the curvature scalar, $\mathcal{G}_{\alpha \beta}$ are the Yang-Mills
field strengths, and $g$ is given in terms of $\varphi$ which is a general function of $z^a$ and $z^{aT}$:

$$g = 3 \ln(- \frac{1}{4} k^2 \varphi) - \ln(\kappa z^a | g |^2).$$

($g^{aT} z^a$ appearing in Eq. (3) is the inverse of the metric $g_{a \bar{a}}$. In the following we shall limit ourselves to the case where the kinetic energy of the scalar field is normalized so that $\kappa z = - \frac{1}{4} \kappa z_{a \bar{a}}$. This choice corresponds to

$$\varphi(z, z^a) = - \frac{3}{\kappa} \exp(- \frac{1}{2} \kappa z_{a \bar{a}} z^a)$$

and the scalar potential takes the form

$$V = \frac{e}{2} \exp \left( \frac{\kappa}{2} z^a z^{aT} \right) \left[ \frac{\partial^2}{\partial z^{aT}} [\kappa z^a z^{aT} - \frac{3}{2} \frac{1}{g} | g |^2] \right] + \frac{e}{2} \kappa \left( e_{\alpha} (Z_a \beta, T^\alpha Z_\beta) \right)^2. \tag{8}$$

The minimization of the potential requires now

$$\frac{\partial g}{\partial z^a} + \frac{1}{4} \kappa z^{aT} = 0. \tag{9}$$

Use of Eqs. (9) in Eq. (8) gives us the value of the potential at the set of solutions $z_0^a$. One finds

$$V_{\text{min}}(z_0^a, z^{aT}_0) = - \frac{e}{4} \kappa^2 | g |^2 \exp(\frac{1}{2} \kappa z_{0 a \bar{a}} z^{aT}_0). \tag{10}$$

It is now clear that the degeneracy of the vacuum solutions encountered in the global supersymmetry case would be lifted due to $O(\kappa^2)$ corrections to the vacuum energy. These corrections are proportional to $g(z_0^a)$ which takes on three different values for the superpotential of Eq. (3). With neglect of $O(\kappa^2 M^2)$ corrections to Eq. (9), the three solutions for $g(z)$ of Eq. (3) are

$$g(z_0^a) = (0, \frac{1}{2}, \lambda_4 M^3, 5 \lambda_4 M^3). \tag{11}$$

The last solution which corresponds to the SU(5) breaking into SU(3)$\otimes$SU(2)$\otimes$U(1) is the one with the lowest energy according to Eq. (10). The solution

$$V_2 = \frac{e}{2} \exp(\frac{1}{2} \kappa^2 | Z |^2) m^4 (|1 + \frac{1}{2} \kappa^2 | Z |^2 + \frac{1}{4} \kappa^2 B_0 Z^* - \frac{3}{8} \kappa^2 | Z |^2 + B_0) \tag{12}$$

The solution to $\partial V / \partial Z = 0$ at the minimum $Z_0$ and the requirements for the vanishing of the vacuum energy $V(Z) = 0$ yield

$$Z_0 = (\sqrt{2} a + \sqrt{6} b) / \kappa; \quad B_0 = - (2 \sqrt{2} a + \sqrt{6} b) / \kappa; \quad a^2 + 1 = b^2, \tag{13}$$

where $a$ and $b$ take on values such that $ab = - 1$, producing two solutions. Though $a$ priori $g_3$ is independent of $\kappa$, the supergravity interactions make the vacuum expectation value of the singlet field $Z$ to be of the order of the Planck mass. Its fermionic partner is then absorbed by the gravitino making it massive and indicating that the super-Higgs phenomena are occurring. The mass of the gravitino is given by $\sqrt{2} m_3 \exp(2 + \alpha \sqrt{3})$ where $m_3 = \kappa M$. Thus if we want to break supersymmetry in the range of 300 GeV to 1 TeV, one must choose $m \sim 10^{11} - 10^{12}$ GeV.

We now arrive at our basic locally supersym-

metric grand unified model defined by the superpotential which is the sum of Eqs. (3) and the super-Higgs term $g_3(Z) = m^2 (Z + B_0)$,

$$g(z^a, Z) = g_1(z^a) + g_3(Z). \tag{14}$$

Introducing the notation $z^A = (z^a, Z)$, one finds that the scalar potential $V(z^A, Z^A)$ in this case is given by Eq. (8) with $z^a$ replaced by $z^A$, etc. We search for solutions $\partial V / \partial z^A = 0$ for this combined potential. In the full analysis of this problem the $g_3$ term acts as a driving term in the Higgs sector and one finds that the Higgs field $H^a (a = 4, 5)$ de-
velops nonvanishing vacuum values.

Since the cross terms between $g_1$ and $g_2$ in Eq. (6) are order $\kappa m^2 \approx m_\gamma$ (or higher) structures, it should be possible to solve for the conditions for minimizing $V$ in a perturbation analysis starting with Eqs. (4) and (13) as the zeroth-order solution. Introducing the notation

$$ (Z_\gamma^3)_{diss} = M (1 + \epsilon_i)(2,2,2,-3+\epsilon_2, -3-\epsilon_2), $$

$$ U = -\kappa m^2 x_\gamma \sqrt{2\lambda_3}, $$

$$ H^c = H^c_\gamma = \gamma \kappa m^2 \sqrt{2\lambda_3} \delta_5^c, $$

we find to lowest order

$$ \epsilon_i = -\kappa m^2 x_\gamma \sqrt{2\lambda_3} M, $$

$$ \epsilon_2 = -\kappa^2 m_\gamma ^2 y_\lambda \lambda_3 / 20\lambda_1 \lambda_3^2 M. $$

$x$ and $y$ obey the algebraic equations

$$ x^2 + y^2 (3 + ab\sqrt{3} - 3\lambda_1) + y^2 + (1 - 3\lambda)^2 = 0, $$

$$ y^2 (3 + ab\sqrt{3} - 3\lambda_1) + 2\kappa y^2 + x = 0, $$

where $\lambda = \lambda_2 / \lambda_1$. One may show that there exists a range of values of $\lambda$ for which Eqs. (18) possess real roots for $x$ and $y$ implying nonvanishing vacuum expectation values of $U$, $H^c$, and $H^c_\gamma$. It is important to note that this breaking of $SU(2) \otimes U(1)$ and supersymmetry produced by supergravity is of $m_\gamma^2$ and hence "semigravitational." (Recall that the Newtonian gravitational constant is $\sim \kappa^2$.) From Eq. (16), then, $\kappa m^2 \sim 300$ GeV to account correctly for the $W$ and $Z$ mass.

Unlike the global supersymmetry case, there are no light scalar bosons in this theory. Those scalar bosons that do not become superheavy gain a common mass $\sim m_\gamma$ from the supersymmetry breaking. The boson partners of quarks, however, have a cancellation of this $m_\gamma^2$ in mass differences, so that these boson mass-squared differences are the order of the corresponding quark mass-squared differences. This leads to a suppression of the flavor-changing neutral currents as in the corresponding globally supersymmetric theory, though note that here the supersymmetry breaking and corresponding boson mass-difference formulas are a consequence of the supergravity model and not put in by hand.

The theory predicts a gravitino mass of $\sim 10^9 - 10^8$ GeV, which is consistent with the recent analysis of Weinberg regarding cosmological constraints on the scale of supersymmetry breaking. The fermionic partners of the Higgs bosons also grow masses $\sim m_\gamma$, indicating a rich array of phenomena in the teraelectronvolt region. The fermionic partners of the $W$ and $Z$ mesons grow masses at the tree level. The only light particles in the theory at the tree level are the fermionic partners of the gluons and the photon.

As is well known, supergravity coupled with matter is not a renormalizable theory in the conventional sense. An approach such as that of "asymptotic safety," however, could possibly make the quantum theory meaningful.

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9The idea that supergravity couplings will break the degeneracy of the different gauge vacua has been independently proposed by S. Weinberg, to be published, and also by D. Ovrut and J. Wess (private communication). The first of the two models treated below is identical to the one considered by Weinberg. Weinberg also shows that each of the resultant nondegenerate vacua are actually stable against bubble formation and hence the physically required $SU(2) \otimes SU(2) \otimes U(1)$ vacuum need not be the one of lowest energy. The stability analysis of Weinberg uses the results of S. Coleman and F. DeLuccia, Phys. Rev. D 21, 3505 (1980).
10A. Salam and J. Strathdee, Phys. Rev. D 11, 1521 (1975).
11The $\lambda_3$ term, introduced in Ref. 5, includes an additional singlet $U$.
12It is possible to construct a more complicated model where the physical ground state corresponding to the $SU(3) \otimes SU(2) \otimes U(1)$ breaking and a vanishing cosmological constant is also the state of lowest energy [P. Nuth, R. Arnowitt, and A. H. Chamseddine, Northeastern University Report No. NUB 2565 (unpublished)]. Here one would not need the Weinberg stability argument.
14A full phenomenological analysis of models with
Quasifree \((e,e'p)\) Reaction on \(^3\text{He}\)

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The proton momentum distribution of \(^3\text{He}\) has been determined up to momenta of 310 MeV/c by use of the reaction \(^3\text{He}(e,e'p)\). The experimental missing-energy resolution, \(\delta E_m = 1.2\) MeV, was sufficient to separate the two- and three-body breakup channels. Results for the three-body disintegration have been obtained up to missing-energy values of 80 MeV. The resulting spectral function is compared to the predictions of Faddeev and variational calculations.

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The theoretical progress made in the last decade in the field of nucleon-nucleon interactions and in the calculation of properties of the three-nucleon system with realistic NN forces\(^1\)\(^,\)\(^2\) has motivated detailed experimental investigations of such systems. Proton knockout coincidence experiments induced by electrons, performed in the region of quasifree kinematics, allow a direct determination of the proton momentum distribution\(^3\) and can therefore serve as a particularly stringent test of NN interaction models.

In the plane-wave impulse approximation (PWIA) the quasifree scattering process is described as follows: An incident electron is scattered elastically from a moving bound target proton with momentum \(\vec{p}\), which is ejected and propagates...