Determining the Dark Matter Relic Density in the mSUGRA $\tilde{\tau}$-$\tilde{\chi}^0_1$ Co-Annihilation Region at the LHC

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We examine the stau-neutralino co-annihilation mechanism of the early universe. We use the minimal supergravity (mSUGRA) model and show that from measurements at the LHC one can predict the dark matter relic density with an uncertainty of 6% with 30 fb$^{-1}$ of data, which is comparable to the direct measurement by WMAP. This is possible by introducing measurements involving b-jet jets to determine the mSUGRA parameters $A_0$ and $\tan\beta$ without direct measurements of the stop and sbottom masses. Our methods provide precision mass measurements of the gauginos, squark, and lighter stau without the mSUGRA assumption.

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One of the important aspects of supersymmetry (SUSY), particularly when it is combined with supergravity grand unification (SUGRA GUT) [1, 2] is that it resolves a number of the problems inherent in the Standard Model (SM). Thus, aside from solving the gauge hierarchy problem and predicting grand unification at the GUT scale $M_G \sim 10^{16}$ GeV, which was subsequently verified at LEP [3], SUGRA GUT allows for the spontaneous breaking of SUGRA at the $M_G$ scale in a hidden sector, leading to an array of soft breaking masses. The renormalization group equations (RGEs) then show that this breaking of SUGRA leads naturally to the breaking of SU(2)$\times$U(1) of the SM at the electroweak scale. This gives rise to an array of SUSY particles accessible at the Large Hadron Collider (LHC) and contributes to a variety of phenomena testable at the electroweak scale, e.g., the anomalous muon magnetic moment $g-2$ (which currently shows a $3.3\sigma$ effect [4]).

An additional feature of SUSY is that models with R-parity invariance automatically give rise to a cold dark matter (CDM) candidate, which is generally the lightest neutralino ($\tilde{\chi}^0_1$), implying a close connection between particle physics and early universe cosmology. The LHC should be able to produce the neutralino, and study its properties. Direct detection of Milky Way DM should allow us to determine the DM mass and its nuclear cross section. If these are in agreement with the LHC determination of the $\tilde{\chi}^0_1$ properties, it would help confirm the important point that the Milky Way DM was indeed the $\tilde{\chi}^0_1$. However, this would not verify explicitly that the $\tilde{\chi}^0_1$ was the DM relic particle produced during the Big Bang. To do this, one would need to deduce the relic density $\Omega h^2$, as measured astronomically by WMAP [5] and compare with the one at the LHC.

In this Letter we describe a series of measurements in the stau-neutralino ($\tilde{\tau}_1$-$\tilde{\chi}^0_1$) co-annihilation (CA) region. This is the region where in the early universe the $\tilde{\tau}_1$ and the $\tilde{\chi}^0_1$ annihilate together into SM particles leaving the correct amount of relic DM abundance. We show how to measure the sparticle masses, confirm we are in the CA region, measure the SUSY parameters and establish a prediction of $\Omega h^2$. The methods introduced are general and can be applied to other regions of SUGRA parameter space. To carry out this analysis, it is necessary to assume a model which encompasses both LHC phenomena and early universe physics. We choose here the simplest example of SUGRA (mSUGRA) [1] which depends on only one sign and four additional parameters: $m_0$ (universal sfermion mass), $m_{1/2}$ (universal gaugino mass), $A_0$ (universal soft breaking trilinear coupling constant), $\tan\beta = (H_1)/(H_2)$, where $(H_1(2))$ is the Higgs vacuum expectation value which gives rise to the up (down) quark masses, and the sign of $\mu$ (the bilinear Higgs coupling constant). Measuring all parameters allow us to calculate $\Omega h^2$. The allowed mSUGRA parameter space with $\mu > 0$, after we include all experimental constraints, has three distinct regions picked out by the CDM constraints [6]: (i) the CA region where both $m_0$ and $m_{1/2}$ can be small, (ii) the focus region where the $\tilde{\chi}^0_1$ has a large Higgsino component and $m_0$ is very large but $m_{1/2}$ is small, and (iii) the funnel region where both $m_0$ and $m_{1/2}$ are large and the neutralinos can annihilate through heavy Higgs bosons ($2M_{\tilde{\chi}^0_1} \sim M_{A_0,H_0}$). We note that a bulk region (where none of the above properties hold) is now almost ruled out due to other experimental constraints.

We consider here the CA region with $\mu > 0$. We use $\mu > 0$ as preferred by measurements of the $b \rightarrow s\gamma$ decay branching ratio and the muon $g-2$ [4]. This region is generic for a wide class of SUGRA GUT models (with or without gaugino universality). Further, if the muon $g-2$ anomaly maintains, then the other two regions are essentially eliminated.

The CA region has a striking characteristic of the $\tilde{\tau}_1$ and $\tilde{\chi}^0_1$ being nearly degenerate i.e., $\Delta M \equiv M_{\tilde{\tau}_1} - M_{\tilde{\chi}^0_1} \sim (5-15)$ GeV. The existence of this near degeneracy would be a strong indication that we are in the CA region. This small $\Delta M$ value is experimentally characterized by a low energy $\tau$ in the $\tilde{\tau}_1 \rightarrow \tau \tilde{\chi}^0_1$ decay. It has been recently shown [7, 8] that the $\Delta M$ value can be measured at the
In order to determine \( \Omega_{\chi_1^0} h^2 \), one must know all mSUGRA parameters. However, measuring \( A_0 \) and \( \tan \beta \) requires the identification of the final states arising from the third generation sparticles, such as stops (\( \tilde{t}_1, \tilde{t}_2 \)), sbottoms (\( \tilde{b}_1, \tilde{b}_2 \)), and staus (\( \tilde{\tau}_1, \tilde{\tau}_2 \)). A technique to determine the four mSUGRA parameters in the bulk region has been developed by identifying the key gluino (\( \tilde{g} \)) and squark (\( \tilde{q} \)) decays, \( \tilde{g} \rightarrow \tilde{b}_1 \tilde{t}_1 \) and \( \tilde{q}_L \rightarrow q \chi_{2}^0 \rightarrow q \ell \ell \chi_{1}^0 \) [9]. Here \( \ell \) is an electron or a muon. However, it is already known that the determination of stop and sbottom masses separately is very difficult if both \( \tilde{g} \rightarrow \tilde{b}_1 \tilde{t}_1 \) and \( \tilde{g} \rightarrow \tilde{b}_1 \tilde{t}_1 \) can occur [10]. The \( \tilde{g} \rightarrow \tilde{t}_1 \tilde{t}_1 \) decay will be a major background for the \( b \) mass measurement. This is also the case for the CA region. However, this technique cannot be applied for the CA case, because the \( \chi_2^0 \rightarrow \ell \ell \chi_1^0 \) decay is essentially absent.

We show that it is indeed possible to determine all four parameters accurately from measurements at the LHC once we introduce a new variable involving \( b \) quarks. The procedure of extracting the model parameters is general and can be applied to other regions of the parameter space or to more general SUGRA models. We then show how to measure (a) the masses of \( \tilde{g}, \tilde{q}_L, \chi_2^0, \chi_1^0 \), and \( \tilde{\tau}_1 \) in the case of \( M_{\chi} \approx M_{\tilde{g}} \gg M_{\tilde{\tau}} \chi_{0}^\pm \) without the mSUGRA assumption and (b) the mSUGRA parameters. We then predict \( \Omega_{\chi_1^0} h^2 \), which can be compared with the astronomical determination of \( \Omega_{CDM} h^2 \).

For our analysis, we select an mSUGRA reference point, shown in Table I, where the total production cross section at the LHC is 9.1 pb. Gluinos and squarks are dominantly produced at the LHC, where \( \tilde{g} \tilde{q} \) production has the largest cross section. The decay chain \( \tilde{q}_L \rightarrow q \chi_2^0 \rightarrow q \tau \tilde{\tau}_1 \rightarrow q \tau \tau \chi_1^0 \) is a characteristic feature of the CA region. The branching ratios for the \( \chi_2^0 \) and \( \tilde{\tau}_1 \) decays are almost 100%. We analyze events in the final state of large transverse missing energy (\( E_T \)) along with jets (\( j \)'s) and \( \tau \) 's.

Events are generated using \texttt{ISAJET} [11], followed by a detector simulation \texttt{PGS4} [12]. We assume the \( \tau \) identification efficiency with \( p_T^{vis} > 20 \) GeV is 50\%, while the probability for a jet being mis-identified as \( \tau \) jet is 1%. We prepare three samples, (i) \( 2\tau + 2j + E_T \), (ii) \( 4\tau + E_T \) and (iii) \( 1b + 3j + E_T \) to measure the masses and the mSUGRA parameters.

The primary SM backgrounds for the \( 2\tau + 2j + E_T \) final state is from \( W+j \) and \( Z+j \) events. For the \( 2\tau + 2j + E_T \) is selected using the following cuts: (a) \( N_\tau \geq 2 \) (\(|\eta|<2.5, p_T^{vis}>20 \) GeV; \( >40 \) GeV for the leading \( \tau \)); (b) \( N_j \geq 2 \) (\(|\eta|<2.5, E_T>100 \) GeV); (c) \( E_T>180 \) GeV and \( E_{T}^{j1}+E_{T}^{j2}+E_{T}>600 \) GeV; (d) veto the event if any of the two leading jets are identified as \( b \). In order to detect \( \chi_2^0 \rightarrow \tau \tilde{\tau}_1 \rightarrow \tau \tau \chi_1^0 \), we categorize all pairs of taus into opposite sign (OS) and like-sign (LS) combinations, and then take the OS minus LS (OS–LS) distributions to effectively reduce the SM events as well as combinatorial SUSY backgrounds. We reconstruct the chain of the cascade decay of \( \tilde{q}_L \) by using the following five kinematical variables: (1) the slope, \( \alpha \), of the \( p_T^{vis} \) distribution for the lower energy \( \tau \) in the OS–LS di-\( \tau \) pairs, (2) the peak position \( M_{\chi}^{peak} \) [13] of the visible di-\( \tau \) invariant mass distribution, (3) the invariant \( j-\tau-\tau \) mass, \( M_{j\tau\tau} \), (4&5) the invariant \( j-\tau \) mass \( M_{j\tau} \) where each of the OS–LS di-\( \tau \) pair is examined separately. With the \( 4j+E_T \) sample, we calculate another variable, \( M_{eff} \equiv E_T + \sum_{4\;jets} E_T^{j} \) [14], which is a function of only the \( \tilde{g} \) and \( q \) masses, using the following cuts (these selection cuts and the SM backgrounds are discussed in Ref. [14]): (a) \( N_{\tau} \geq 4 \) (\(|\eta|<2.5, E_T>100 \) GeV for the leading jet; \( >50 \) GeV for other jets); (b) \( E_T>100 \) GeV; (c) Transverse sphericity \( >0.2 \); (d) Vetoing isolated electrons or muons with \( p_T>15 \) GeV and \(|\eta|<2.5 \). Again we require that none of these jets identified as a \( b \) jet.

Using the third sample \( 1b + 3j + E_T \), we introduce a new variable, \( M_{eff}^{(b)} \), similar to \( M_{eff} \), but requiring that the leading jet to be a \( b \) jet.

The measurement of a small value of \( \alpha \) from the \( 2\tau + 2j + E_T \) indicates low energy \( \tau \)'s in the final state (thus \( \Delta M \) is small) and provides a smoking gun signal of the CA region. In Fig. 1[top], we show the \( p_T^{vis} \) distribution obtained by the OS–LS technique, inspiring a \( \Delta M \) dependence expected in the CA region. It is interesting to note that \( \alpha \) only depends on the \( \tilde{\tau}_1 \) and \( \chi_1^0 \) masses as shown in Fig. 1[bottom].

To complete measurements of the sparticle masses, we use the remaining variables. The variables, \( M_{j\tau\tau} \) and \( M_{j\tau\tau} \), probe \( \tilde{q}_L \rightarrow q \chi_2^0 \rightarrow q \tau \tilde{\tau}_1 \rightarrow q \tau \tau \chi_1^0 \) decay chains. To help identify these chains we additionally require OS–LS di-tau pairs with \( M_{\tau\tau}<M_{\tau\tau}^{end\;point} \) and construct \( M_{j\tau\tau} \) for every jet with \( E_T>100 \) GeV in the event. With three jets, there are three masses: \( M_{j\tau\tau}^{(1)} \), \( M_{j\tau\tau}^{(2)} \), and \( M_{j\tau\tau}^{(3)} \) in a decreasing order. We choose \( M_{j\tau\tau}^{(2)} \) for this analysis [14].

Figures 2 shows the \( M_{j\tau\tau}^{(2)} \) distributions for two different \( \tilde{q}_L \) masses and its peak position \( M_{j\tau\tau}^{peak} \) as a function of \( M_{\tilde{q}_L} \) and \( M_{\chi_1^0} \), keeping \( \Delta M \) constant. Similarly, one can show that the \( M_{j\tau\tau}^{peak} \) value depends on the \( \tilde{q}_L \), \( \chi_2^0 \), \( \tilde{\tau}_1 \) and \( \chi_1^0 \) masses. The peak position of \( M_{eff} \), \( M_{eff}^{(b)} \),

| TABLE I: SUSY masses (in GeV) for our reference point \( m_{1/2} = 350 \) GeV, \( m_0 = 210 \) GeV, \( \tan \beta = 40 \), \( A_0 = 0 \), and \( \mu > 0 \). |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( \tilde{g} \)   | \( \tilde{u}_L \) | \( t_2 \)        | \( b_2 \)        | \( \tilde{c}_L \) | \( \tilde{\tau}_2 \) | \( \tilde{\chi}_2^0 \) | \( \Delta M \)  |
| 731              | 748             | 728             | 705             | 319             | 329             | 260             | 0.3            |
| 725              | 561             | 645             | 251             | 151.3           | 140.7           | 10.6            |                |
has been shown to be a function of only the $\tilde{g}_L$ and $\tilde{g}$ masses [14].

The determination of the sparticle masses using the above variables only is done by inverting the six functional forms (see Ref. [15]) to simultaneously solve for $\tilde{g}$, $\chi_1^0$, $\tilde{t}_1$, and average $\tilde{q}_L$ masses and their uncertainties. With 10 fb$^{-1}$ of data, we obtain (in GeV) $M_{\tilde{g}} = 831 \pm 28$, $M_{\chi_1^0} = 260 \pm 15$, $M_{\chi_0^0} = 141 \pm 19$, $\Delta M = 10.6 \pm 2.0$, $M_{\tilde{q}_L} = 748 \pm 25$. The accurate determination of $\Delta M$ confirms that we are in the CA region. We also test the universality of gaugino masses at the GUT scale which implies $r_1 = M_{\tilde{g}}/M_{\chi_1^0} = 5.91$ and $r_2 = M_{\tilde{g}}/M_{\chi_0^0} = 3.19$ at the electroweak scale. With the above gaugino masses, we obtain $r_1 = 5.9 \pm 0.8$ and $r_2 = 3.1 \pm 0.2$, validating the universality relations to 14% and 6%, respectively.

Since our primary goal is to determine $\Omega_{\chi_1^0}h^2$ in the mSUGRA model, we determine $m_0$ and $m_{1/2}$ using $M_{\text{eff}}$ and $M_{\tilde{t}_{\tilde{t} \tilde{t}}}$, while $A_0$ and $\tan \beta$ are most sensitive to $M_{\tilde{t}_{\tilde{t}}}$ and $M_{\tilde{q}_L}$. $M_{\tilde{q}_L}$ and $M_{\tilde{t}_{\tilde{t} \tilde{t}}}$ depend only on $\tilde{q}_L$ (first two generations), $\tilde{g}$, $\chi_0^0$ and $\tilde{g}$ masses. In Fig. 3 we show the dependences of the peak positions of these variables on $m_0$ and $m_{1/2}$. Thus, these peak positions can be used to determine $m_0$ and $m_{1/2}$ without requiring any knowledge of $A_0$ or $\tan \beta$. On the other hand, $M_{\tilde{t}_{\tilde{t}}}$ and $M_{\tilde{q}_L}$ depend on the $\tilde{t}_1$ and the $\tilde{b}_1$ masses, respectively. Since $\tilde{t}_1$ and $\tilde{b}_1$ decays always produce at least one $b$ jet in the final state, it can be related to $\tilde{t}_1$ and $\tilde{b}_1$ masses. We plot $M_{\tilde{t}_{\tilde{t}}}$ and $M_{\tilde{q}_L}$ as functions of $A_0$ and $\tan \beta$ in Fig. 4.

Combining these four measurements we determine $m_0$, $m_{1/2}$, $A_0$ and $\tan \beta$ with small statistical uncertainties: $m_0 = 205 \pm 4$ GeV; $m_{1/2} = 350 \pm 4$ GeV; $A_0 = 0 \pm 16$ GeV; $\tan \beta = 40 \pm 1$ with 10 fb$^{-1}$ of data. This procedure of determining the mSUGRA parameters is general and can be applied to other SUGRA models.

After measuring the mSUGRA variables, we calculate $\Omega_{\chi_1^0}h^2$ using Darksusy [16]. In the CA region, $\Omega_{\chi_1^0}h^2$ depends crucially on $\Delta M$ due to the Boltzmann suppression factor $e^{-\Delta M/k_B T}$ in the relic density formula [17]. Figure 5 shows contour plots of the 1σ uncertainty in $\Omega_{\chi_1^0}h^2$.
ΔM plane. The uncertainty on Ωχ01h2 is 11 (4.8)% at 10 (50) fb⁻¹. Note that it is 6.2% at 30 fb⁻¹, comparable to that of the WMAP measurement [5].

In conclusion, we have established a technique for a precision measurement of Ωχ01h2 using the τ₁χ01 CA region of the mSUGRA model at the LHC. This is done using only the model parameters, determined by the kinematical analyses of 3 samples of $E_T + j's + \tau's$ events with and without b jets. The accuracy of the Ωχ01h2 calculation at 30 fb⁻¹ of data is expected to be comparable to that of Ω CDM h² by WMAP. With measurements at the LHC, it is possible to confirm that the DM we observe today were χ01's created in the early universe.

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[9] G. Polesello and D. R. Tovey, JHEP 0405, 071 (2004); M. M. Nojiri, G. Polesello and D. R. Tovey, JHEP 0603, 063 (2006).
[12] PGS is a parametrized detector simulator. We use the CMS detector configuration to find gluon/light quark jets and b jets. A parametrized b-tagging efficiency is used based on the CDF SECVTX b-tagger in A. Abulencia et al., Phys. Rev. Lett. 97, 082004 (2006). We used version 4 (see http://www.physics.ucdavis.edu/~conway/research/software/pgs/pgs4-general.htm). We take the b-tagging efficiency to be ~42% for b jets with $E_T > 50$ GeV and $|\eta| < 1.0$, and degrading between 1.0 $< |\eta| < 1.5$. The tagging fake rate for c and light quarks/gluons is ~9% and 2% respectively.
[13] The end point of the Mττ distribution depends upon $\bar{\tau}_1$ and $\chi^0_{1,2}$ masses, which is an established technique in the study of selectron and smuon decays [14]. The end point in the stau case is challenging because of the escaping neutrinos from the $\tau$ decays. Thus, a precise end point measurement requires a full understanding of the $\tau$ energy resolution and the shape of the background near the end point. Therefore we choose the peak position since it is less sensitive to those concerns.
[15] Each of the following six variables is parametrized as a function of SUSY masses: (1) $M_{\tau\tau}^{peak} = f_1(M_{\tilde{q}0}, M_{\tilde{t}0}, X_1, X_2, \Delta M)$, (2) $\alpha = f_2(M_{\tilde{q}0}, \Delta M)$, (3) $M_{j\tau\tau}^{peak} = f_3(M_{\tilde{q}0}, M_{\tilde{g}}, M_{\tilde{t}0} - M_{\tilde{g}}, M_{\tilde{t}0} - M_{\tilde{g}})$, (4&5) $M_{j\tau\tau}^{peak} = f_4(M_{\tilde{q}0}, M_{\tilde{t}0} - M_{\tilde{g}}, M_{\tilde{t}0} - M_{\tilde{g}})$. $M_{\tau\tau}^{peak}$, $M_{j\tau\tau}^{peak}$. $M_{\tau\tau}^{peak}$, $M_{j\tau\tau}^{peak}$.
\[ \Delta M \), \( M_{\text{peak}}^{\text{eff}} = f_6(M_{\tilde{q}_L}, M_{\tilde{q}}). \]
