Chapter 5

Kinematics for \( \tilde{\chi}_2^0 \) searches

At the Large Hadron Collider the colliding parton’s center of mass and exact center of mass energy is not known. Therefore, it is sensible to concentrate on Lorentz invariant quantities like the invariant mass. The invariant mass of \( n \) particles with energy \( E_i \) and 3-momentum \( p_i \) is frame independent, and given by

\[
M^2 = \left( \sum_{i=1}^{n} E_i \right)^2 - \left( \sum_{i=1}^{n} p_i \right)^2
\] (5.1)

The lowest limit of \( M \) is in any case the sum of all \( n \) particle rest frame masses. If there is no external force the invariant mass of a system of particles does not change, even if some particles decay or annihilate. In the following calculations the speed of light \( c \) and Planck’s constant \( \hbar \) are set to one.

Invariant mass distributions carry information about the particles at the end of a decay chain as well as about the decaying particles. For this study, the kinematic formulae and configurations presented in this chapter are relevant. In general it is necessary to distinguish between massive and massless particles. If the energy of a particle is a few magnitudes higher than its mass, it is a good estimation to treat it as massless. Therefore, in the following calculations [20] all Standard Model quarks and leptons - except the top quark in section 5.3 - are chosen to be massless. In the Monte Carlo simulations, however, the correct masses are taken into account.

5.1 Sequential two-body decays

Only two-body decays have to be taken into account for the processes in this study. In the two-body decay of a particle \( M \) into \( m_1 \) and \( m_2 \), the energy, \( E_i^* \), and 3-momentum, \( p_i^* \), in the rest frame of \( M \), which is denoted with a “*”, are given by

\[
M = E_1^* + E_2^*, \quad |p_1^*| = |p_2^*| = p^*.
\] (5.2)

Squaring the first equation gives

\[
2E_1^*E_2^* = M^2 - m_1^2 - m_2^2 - 2p^2.
\]

Squaring again and extracting \( p^* \) leads to

\[
p^* = \frac{1}{4M^2} [(M^2 - m_1^2 - m_2^2)^2 - 4m_1^2m_2^2]
\]

\[
= \frac{1}{4M^2} [M^4 - 2M^2(m_1^2 + m_2^2) + (m_1^2 - m_2^2)^2]
\]

\[
= \frac{1}{4M^2} [M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2].
\]
The energy becomes
\[ E_1^* = p^* + m_1^2 \]
\[ = \frac{1}{4M^2} [M^4 - 2M^2(m_1^2 + m_2^2) + (m_1^2 - m_2^2)^2 + 4M^2m_1^2] \]
\[ = \frac{1}{4M^2} [M^2 + m_1^2 - m_2^2]^2. \]

In summary:
\[ E_1^* = \frac{M^2 + m_1^2 - m_2^2}{2M} \] (5.3)
\[ |p_1^*| = |p_2^*| = \frac{\left( (M^2 - (m_1 + m_2)^2)(M^2 - (m_1 - m_2)^2) \right)^{1/2}}{2M} \] (5.4)

The formulae are considerably simplified if one of the decay particles is massless. Assuming that \( m_2 = 0 \) one obtains:
\[ E_1^* = \frac{M^2 + m_1^2}{2M}, \quad E_2^* = \frac{M^2 - m_1^2}{2M}, \quad |p_1^*| = |p_2^*| = \frac{M^2 - m_1^2}{2M}. \] (5.5)

The transformation from the rest frame of \( M \) to the lab system is defined by two kinematic variables of \( M \), the velocity \( \beta = q/E \) and \( \gamma = E/M \), where \( q \) and \( E \) are the lab frame momentum and energy of \( M \). The Lorentz transformation to the lab system is most easily decomposed into a longitudinal and a transverse part:
\[ E_i = \gamma [E_i^* + \beta p^* \cos \theta_i^*] \]
\[ p_{Ti} = p_{T1}^* = p^* \sin \theta_i^* \]
\[ p_{Li} = \gamma [\beta E_i^* + p^* \cos \theta_i^*] \] (5.6)

with \( \theta_i^* = \theta_i^* + \pi \). The maximum and minimum values of \( E_i \) and \( p_{Li} \) are obtained for \( \cos \theta_i^* = +1 \) and \( -1 \) respectively.

The investigated leptonic \( \tilde{\chi}_0 \) decay chain in the analysis presented here is
\[ \tilde{\chi}_0^0 \rightarrow \tilde{\ell}^\pm + \ell^\mp, \quad \tilde{\ell}^\pm \rightarrow \tilde{\chi}_1^0 + \ell^\pm. \] (5.7)

The calculations are valid for taus as well as for other leptons \( l \), which are assumed to be massless. Moreover, maxima and minima of effective masses correspond to collinear configurations, where the particles are emitted along or opposite to the direction of the Lorentz boost. In this case,
\[ E_{l2} = \gamma E_{l2}^* (1 \pm \beta). \] (5.8)

As the Lorentz transformation parameters are
\[ \beta = \frac{M_{\tilde{\chi}_0}^2 - M_{\tilde{l}}^2}{M_{\tilde{\chi}_0}^2 + M_{\tilde{l}}^2}, \quad \gamma = \frac{M_{\tilde{\chi}_0}^2 + M_{\tilde{l}}^2}{2M_{\tilde{\chi}_0}M_{\tilde{l}}}, \] (5.9)
this yields to
\[ \gamma (1 \pm \beta) = \frac{1}{2M_{\tilde{\chi}_0}M_{\tilde{l}}} \left[ (M_{\tilde{\chi}_0}^2 + M_{\tilde{l}}^2) \pm (M_{\tilde{\chi}_0}^2 - M_{\tilde{l}}^2) \right] \] (5.10)

so that the maximum energies associated with the boost direction and minimum energies associated with the opposite-to-the-boost direction are:
\[ E_{l2}^{\text{max}} = \frac{M_{\tilde{\chi}_0}}{M_{\tilde{l}}} E_{l2}^*, \quad E_{l2}^{\text{min}} = \frac{M_{\tilde{l}}}{M_{\tilde{\chi}_0}} E_{l2}^*. \] (5.11)
Similarly, for the momentum with the sign being measured in the boost direction it is

\[ p_{l2} = \gamma E_{l2}^* (\beta \pm 1) \]  

leading to

\[ p_{l2}^{\text{max}} = E_{l2}^{\text{max}}, \quad p_{l2}^{\text{min}} = -E_{l2}^{\text{min}}. \]  

These expressions considerably simplify the later calculations. In the following the kinematic limit for both leptons is calculated. The effective mass can be computed in the rest frame of \( \tilde{\chi}_2^0 \).

The z-axis is chosen along the direction of the slepton, as depicted on figure 5.1.

![Image](image_url)  

Figure 5.1: Kinematics of the \( \chi_2^0 \) decay via a slepton \( \tilde{l} \).

The dileptonic effective mass is given by

\[ M_{ll}^2 = (E_{l1} + E_{l2})^2 - (-p_{l1} + p_{lLl2})^2 - (0 + p_{lTl2})^2 \]

\[ = 2E_{l1}E_{l2} + 2p_{l1}p_{lLl2} = 2E_{l1}(E_{l2} + p_{lLl2}) \]  

where the indices \( L \) and \( T \) denote the longitudinal and transverse components. As particle \( l1 \) is massless, the energy and momentum of \( l1 \) and \( \tilde{l} \) are given by the expressions and similarly for particle \( l2 \) in the rest frame of \( \tilde{l} \).

\[ E_{l2}^* = |p_{l2}^*| = \frac{M_{l}^2 - M_{\tilde{\chi}_2^0}^2}{2M_{\tilde{l}}} \]  

The Lorentz transformation of the massless particle \( l2 \) into the rest frame of \( \tilde{\chi}_2^0 \) becomes

\[ E_{l2} = \gamma E_{l2}^*(1 + \beta \cos \theta^*) \]

\[ p_{lLl2} = \gamma E_{l2}^*(\beta + \cos \theta^*) \]  

from which

\[ E_{l2} + p_{lLl2} = \gamma E_{l2}^*(1 + \beta)(1 + \cos \theta^*) \]

\[ = \frac{M_{\tilde{\chi}_2^0}^2}{2M_{\tilde{l}}^2}(M_{l}^2 - M_{\chi_1^0}^2)(1 + \cos \theta^*). \]  

The effective mass then takes the simple expression

\[ M_{ll}^2 = \frac{M_{\chi_2^0}^2 - M_{\tilde{l}}^2 M_{\tilde{\chi}_2^0}^2}{2M_{\chi_2^0}^2} \frac{M_{l}^2 - M_{\chi_1^0}^2}{2M_{\tilde{l}}^2}(1 + \cos \theta^*) \]

\[ = \frac{(M_{\chi_2^0}^2 - M_{\tilde{l}}^2)(M_{l}^2 - M_{\chi_1^0}^2)}{2M_{l}^2}(1 + \cos \theta^*) \]  

19
with the upper endpoint of the mass distribution given by

\[
M^\text{max}_{ll} = M_{\tilde{\chi}_2^0} \sqrt{\left(1 - \frac{M_l^2}{M_{\tilde{\chi}_2^0}^2}\right) \left(1 - \frac{M_{\tilde{\chi}_1^0}^2}{M_l^2}\right)}.
\] (5.19)

This corresponds to the configuration where the two leptons are emitted back-to-back in the rest frame of the $\tilde{\chi}_2^0$. Since the $\tilde{l}$ is a particle with spin 0 it should decay isotropically. Thus, formula (5.18) shows that the distribution in $M_{ll}$ increases linearly with $M_{ll}$ leading to a sharp edge at the kinematic limit given by formula (5.19).

The following section shows a summary of all formulae for the endpoints which are used in the analysis and the related configurations. They are calculated in the same way as described above.

### 5.2 Formulae and kinematic configurations

Here the endpoints available for the decay chain $\tilde{q} \rightarrow q + \tilde{\chi}_2^0$, $\tilde{\chi}_2^0 \rightarrow l1 + \tilde{l}$, $\tilde{l} \rightarrow l2 + \tilde{\chi}_1^0$ are summarized.

1. $M^\text{max}_{ll}$

\[
M^\text{max}_{ll} = M_{\tilde{\chi}_2^0} \sqrt{\left(1 - \frac{M_l^2}{M_{\tilde{\chi}_2^0}^2}\right) \left(1 - \frac{M_{\tilde{\chi}_1^0}^2}{M_l^2}\right)}
\] (5.20)

![Diagram of $M^\text{max}_{ll}$](image)

2. $M^\text{max}_{l1q}$ (1st lepton)

\[
M^\text{max}_{l1q} = M_{\tilde{q}} \sqrt{\left(1 - \frac{M_{\tilde{\chi}_2^0}^2}{M_{\tilde{q}}^2}\right) \left(1 - \frac{M_{\tilde{l}}^2}{M_{\tilde{\chi}_2^0}^2}\right)}
\] (5.21)

![Diagram of $M^\text{max}_{l1q}$](image)
3. $M_{l_2q}^{\text{max}}$ (2nd lepton)

$$M_{l_2q}^{\text{max}} = M_{\tilde{q}} \sqrt{ \left( 1 - \frac{M_{\chi_0^0}^2}{M_{\tilde{q}}^2} \right) \left( 1 - \frac{M_{\chi_0^0}^2}{M_{\tilde{l}}^2} \right) }$$  \hspace{1cm} (5.22)

4. $M_{l_1q}^{\text{max}}$, for $M_{\chi_0^0}^2 < M_{\tilde{q}} M_{\chi_1^0}$

$$M_{l_1q}^{\text{max}} = M_{\tilde{q}} \sqrt{ \left( 1 - \frac{M_{\chi_0^0}^2}{M_{\tilde{q}}^2} \right) \left( 1 - \frac{M_{\chi_0^0}^2}{M_{\chi_1^0}^2} \right) }$$ \hspace{1cm} (5.23)

5. $(M_{l_1q} + M_{l_2q})^{\text{max}}$

$$M_{l_1q + l_2q}^{\text{max}} = M_{l_1q}^{\text{max}} + \frac{M_l}{M_{\chi_0^0}} M_{l_2q}^{\text{max}}$$ \hspace{1cm} (5.24)
Four of these five endpoints are in principle necessary to determine all involved masses, provided that several consecutive decay channels are open (long decay chains). However, it will not always be possible to measure them precisely. Therefore, it will be sensible to use all available endpoints.

### 5.3 Endpoints for top quark events

The mass of the top quark is very large and therefore not negligible. Thus, not only upper endpoints but also lower endpoints arise in the invariant mass distributions. In the analysis presented in section 6.2.13 to 6.2.15 the top quark is involved. The corresponding decay chain is

\[
\tilde{t} \rightarrow t + \tilde{\chi}_2^0 \rightarrow t + \tilde{t} + l \rightarrow t + l + l + \tilde{\chi}_1^0. \tag{5.25}
\]

With

\[
E_t = \frac{M_t^2 + m_t^2 - M_{\tilde{\chi}_2}^2}{2M_t}, \quad p_t = \sqrt{E_t^2 - m_t^2} \tag{5.26}
\]

\[
E_{\tilde{\chi}_2} = \frac{M_t^2 + M_{\tilde{\chi}_2}^2 - m_t^2}{2M_t}, \quad p_{\tilde{\chi}_2} = \sqrt{E_{\tilde{\chi}_2}^2 - M_{\tilde{\chi}_2}^2} \tag{5.27}
\]

the endpoint formulae are the following:

\[
(M_{1t}^{\text{max}})^2 = m_t^2 + M_t^2(1 - \frac{M_{\tilde{\chi}_2}^2}{M_t^2}) \left(1 - \frac{M_{\tilde{\chi}_2}^2}{M_{\tilde{\chi}_2}^2} \right) \frac{E_{\tilde{\chi}_2} + p_{\tilde{\chi}_2}}{M_t} \tag{5.28}
\]

\[
\times \frac{1}{2} \left( 1 + \sqrt{1 - \frac{m_t}{E_t}}^2 \right)
\]

\[
(M_{1t}^{\text{min}})^2 = m_t^2 + M_t^2(1 - \frac{M_{\tilde{\chi}_2}^2}{M_t^2}) \left(1 - \frac{M_{\tilde{\chi}_2}^2}{M_{\tilde{\chi}_2}^2} \right) \frac{E_{\tilde{\chi}_2} - p_{\tilde{\chi}_2}}{M_t} \tag{5.29}
\]

\[
\times \frac{1}{2} \left( 1 - \sqrt{1 - \frac{m_t}{E_t}}^2 \right)
\]

\[
(M_{2t}^{\text{max}})^2 = m_t^2 + M_t^2(1 - \frac{M_{\tilde{\chi}_2}^2}{M_t^2}) \left(1 - \frac{M_{\tilde{\chi}_2}^2}{M_{\tilde{\chi}_2}^2} \right) \frac{E_{\tilde{\chi}_2} + p_{\tilde{\chi}_2}}{M_t} \tag{5.30}
\]

\[
\times \frac{1}{2} \left( 1 + \sqrt{1 - \frac{m_t}{E_t}}^2 \right)
\]

\[
(M_{2t}^{\text{min}})^2 = m_t^2 + M_t^2(1 - \frac{M_{\tilde{\chi}_2}^2}{M_t^2}) \left(1 - \frac{M_{\tilde{\chi}_2}^2}{M_{\tilde{\chi}_2}^2} \right) \frac{E_{\tilde{\chi}_2} - p_{\tilde{\chi}_2}}{M_t} \tag{5.31}
\]

\[
\times \frac{1}{2} \left( 1 - \sqrt{1 - \frac{m_t}{E_t}}^2 \right)
\]

\[
(M_{lt}^{\text{max}})^2 = m_t^2 + M_t^2(1 - \frac{M_{\tilde{\chi}_2}^2}{M_t^2}) \left(1 - \frac{M_{\tilde{\chi}_2}^2}{M_{\tilde{\chi}_2}^2} \right) \frac{E_{\tilde{\chi}_2} + p_{\tilde{\chi}_2}}{M_t} \tag{5.32}
\]

\[
\times \frac{1}{2} \left( 1 + \sqrt{1 - \frac{m_t}{E_t}}^2 \right)
\]

\[
(M_{lt}^{\text{min}})^2 = m_t^2 + M_t^2(1 - \frac{M_{\tilde{\chi}_2}^2}{M_t^2}) \left(1 - \frac{M_{\tilde{\chi}_2}^2}{M_{\tilde{\chi}_2}^2} \right) \frac{E_{\tilde{\chi}_2} - p_{\tilde{\chi}_2}}{M_t} \tag{5.33}
\]

\[
\times \frac{1}{2} \left( 1 - \sqrt{1 - \frac{m_t}{E_t}}^2 \right)
\]
The configurations for the upper endpoints here are the same as for the massless case described in section 5.2. The lower endpoints are realised with a configuration where all detected particles go into the same direction.