A 2kg rod with a length of 1m and resistance of 4Ω slides with constant velocity down a pair of vertical frictionless wires that are connected to each other. The rest of the circuit is without resistance. A uniform magnetic field of 3T is perpendicular to the plane of the circuit. What is the current and what direction does it flow? What is the speed of the rod? What is the power of energy dissipation through the system?

First off, I need to make sure that this thing can be done. To do that, I need to find the direction of the current and see if it produces and opposing force to gravity in the presence of the magnetic field. Let’s say that the magnetic field is pointing out of the board. The rod is coming down and losing magnetic flux through the loop. A current will be induced in the circuit which will oppose the loss of magnetic flux. The current will produce a magnetic field which is out of the board, so the current must be flowing counter-clockwise. The current flowing counter-clockwise, or from right to left in the bar on top, will produce a force upward. The upward force can cancel gravity, so this setup is possible.

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\[ F_g = F_B \]
\[ mg = LIB \]
\[ mg = L \frac{vBL}{R} B = \frac{v}{R} (BL)^2 \]
\[ v = \frac{mgR}{(BL)^2} \]
\[ v = \frac{2kg(9.8 \text{m/s}^2)}{(3Tm)^2} = 8.71 \text{ m/s} \]
\[ I = \frac{vBL}{R} = \frac{BL \frac{mgR}{(BL)^2}}{R} = \frac{mg}{BL} \]
\[ I = \frac{2kg(9.8 \text{m/s}^2)}{3Tm} = 6.53 \text{A} \]

Now that I have current, electromotive force, velocity, and force, I can combine them in 2 ways to arrive at power and check for consistency.

\[ P = IV = I^2R \]
\[ P = \left( \frac{mg}{BL} \right)^2 R \]
\[ P = Fv \]
\[ P = mg \frac{mgR}{(BL)^2} \]
\[ P = \left( \frac{mg}{BL} \right)^2 R \]
\[ P = \left( \frac{2kg(9.8 \text{m/s}^2)}{3Tm} \right)^2 4\Omega = 171 \text{W} \]

We then want to repeat the problem but instead of closing the loop at the bottom, we want to close the loop at the top. Here, the magnetic flux is increasing and so the induced current will generate a magnetic field to oppose that increase. The generated magnetic field is into the paper. The current to do that is clockwise, which is still from right to left. The current is flowing through the bar in the same direction as before. No other numbers changed. So no other predictions changed. The velocity is still 8.71 m/s, the current is 6.53A, and the power is 171W.

I have a 48V battery connected to a switch and two parallel branches. One branch contains a resistor of 6Ω and the other contains a resistor of 8Ω and an inductor of .5H. When the switch is closed, current begins to flow. We want to know the current and power through each resistor when the switch is closed, when the magnetic field of the inductor is saturated, and when half of the current is flowing through the inductor that flows at magnetic field saturation.
So I’ll start with the current when the switch is closed. The inductors goal in life is to keep current flowing at the same rate all times, or to keep current constant. The current flowing through it after the switch is closed is 0A. The current flowing through $R_1$ is in agreement with Ohm’s law, $I_1 = \frac{48V}{6\Omega} = 8A$. Since no current is flowing through the inductor, its power must be 0W.

When the circuit has reached equilibrium, the current flowing through the inductor and $R_2$ is in agreement with Ohm’s law for the resistor, $I_2 = \frac{48V}{8\Omega} = 6A$. There is no potential drop across the inductor, so its power must be 0W again. The current through the first resistor is unchanged. So the final current being 6A, means that half current flowing through the resistor and inductor is 3A. When 3A flows through an 8Ω resistor, the potential drop across it is 24V leaving 24V for the inductor. The power through both must be the same with the same current and potential, $P = 3A \times 24V = 72W$.

The problem continues from the saturated magnetic field above with a now open battery circuit.

As previously noted, the inductor resists changed in current through it. When the previous problem ended, it had 6A of current running through it. That has not changed. That current now runs through the two resistors in the circuit. The total resistance of the circuit is 14Ω. That means that the potential across the inductor is $V = IR = 6A \times 14\Omega = 84V$. The power of the inductor is $P = IV = 6A \times 84V = 504W$. The power of $R_1$ is $P = I^2R = (6A)^2 \times 6\Omega = 216W$. The power of $R_2$ is 288W. When 3A flow through the inductor, for the same reasons as above, it has a power of 126W and potential of 42V. When all of the energy has been drained out of the inductor, the potential across it is zero volts and the power is zero watts.