Orbits

\[ F_{\text{gravity}} = \frac{G M_1 M_2}{R^2} = m \cdot \frac{V^2}{R} \]

\[ V = \sqrt{\frac{G M}{R}} \]

\[ T = \text{period, time to complete an orbit} \]

\[ V = \frac{2\pi R}{T} \quad \Rightarrow \quad T = \frac{2\pi R}{V} \]

\[ = \frac{2\pi R}{\sqrt{\frac{G M}{R}}} = \frac{2\pi R^{\frac{3}{2}}}{\sqrt{G M \cdot R}} = 2\pi \sqrt{\frac{R^3}{GM}} \]

Kepler's 3rd Law - \( T^2 \propto R^3 \)

\[ T = 2\pi \sqrt{\frac{R^3}{GM}} \quad \Rightarrow \quad T^2 = \frac{4\pi^2 R^3}{GM} = \frac{Y M}{G M} \]

\[ T = 92.69 \text{ min} = 55561.4 \text{ s} \]

\[ M_0 = 5.97219 \times 10^{24} \text{ kg} \]

\[ G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2} \]

\[ T = \frac{2\pi R^{\frac{3}{2}}}{GM} \quad \Rightarrow \quad R^2 = \frac{T^2 GM}{4\pi^2} \]

\[ R = \sqrt{\frac{T^2 GM}{4\pi^2}} \]

\[ = 6.78 \times 10^6 \text{ m} \quad \text{orbital} \quad 6.787 \times 10^6 \text{ m} \]

Energy - The ability to do something
Potential Energy - Energy of position. Atoms lose it as they bond. An eraser gains it over being on the ground when it is on a table. I prefer PE. The boat uses U. I've also seen V.

Kinetic Energy - Energy of motion. Anything moving has it. I prefer KE. The boat uses K. I've also seen T.

Energy can convert between forms. It can move from PE to KE.

Chemical potential energy can become kinetic energy & the mechanical potential potential energy.
Work - Force applied over a distance
\[ W = \int F \cdot ds = F \cdot x = F \cdot r \cos \theta = F_r \cdot r \text{ individual force} \]

\[ F \text{, } r \text{, and } \theta \text{ are constant, thus } \int ds \text{ is straight.} \]

\[ W = F_r \cdot r = [N \cdot m] = [J \text{, joules}] \quad \text{C} = [N \cdot m] \]

Energy is J, torque is Nm to distinguish between the two when only a quantity is given.

\[ W_{\text{tot}} = F_{\text{net}} \cdot r = F_{\text{net}} \cdot \theta \text{ total force } \cdot \text{ total work} \]

\[ = \Delta K = K_E - K_i = \frac{1}{2} m \left( v_f^2 - v_i^2 \right) \]

This statement is known as the Work-Energy Theorem.

Kinetic Energy \[ K = \frac{1}{2} m v^2 \]

not in the class

Example: Distance traveled coming to rest from 50% 99N force is acting on a 7 kg object.

Forces:
\[ F = ma \Rightarrow 99N = 7kg \cdot a \]
\[ a = \frac{99}{7} = 14.1 m/s^2 \]
\[ v_f^2 = v_i^2 + 2ad \]
\[ - \frac{14.1}{2} = -d \left( 50\% \right)^2 \]
\[ \Rightarrow d = 17.9 m \]

\[ \text{Work-Energy Theorem:} \]
\[ \Delta K = F \cdot d \cos \theta \]
\[ \frac{1}{2} \times 7kg \left( 0\% \right)^2 - \frac{1}{2} \times 7kg \left( 50\% \right)^2 = +99N \cdot d \cdot \cos \left( 180\% \right) \]
\[ - \frac{1}{2} \times 7kg \left( 50\% \right)^2 = \frac{99N}{99N \cdot \cos \left( 180\% \right)} \]
\[ = 17.9 m \]
Potential energy describes work done against a conservative force.

PE is only definable for conservative forces. If a potential has been defined $V$ for a force, then the associated force is a conservative force.

Work done by a conservative force doesn't care about the path taken from one point to another.

$$W_{21} = W_{22} \quad \text{All that matters here is that} \quad \theta \text{ is } 2 \text{ blocks higher than } \alpha$$

Potential energy can be negative. If I decide that $B=OJ$, then $\theta$ has negative potential energy. If I decide that $\alpha$ is $OJ$, then $\theta$ has positive PE. The point that matters here is that the PE between the two points is the same in either case.

We have 2 major PE in this class.

**PE gravity** $= mg \cdot h = F_{gravity} \cdot \ell = F \cdot d$

**PE spring** $= k \cdot x^2 = k \cdot (x-x_0)$

Consider a mass hanging on a spring.

$$E_{spring} = -kx$$

$$E_{grav} = -mg$$

$$kx = mg$$

$$x = \frac{mg}{k} = \frac{5 \text{ kg } \times 9.8 \text{ m/s}^2}{2 \text{ N/m}} = 24.5 \text{ m}$$

**Energy wants to minimize**

$$E = \frac{1}{2}kx^2 + mx$$

$$= 100 \text{ N/m} \cdot x^2 + 5 \text{ kg } \times 9.8 \text{ m/s}^2 \cdot x$$

Energy conservation: Energy is universally conserved.

Energy may not be conserved in a system, the exception is when:

- $E$ is constant if there are no non-conservative forces but may be treated.
- $E$ is constant in elastic collisions but non-constant in inelastic collisions.