Measuring forces:

\[ F = \text{Force from object to be measured} \]

\[ F = \text{known force} \]

\[ F = m a \]

\[ a \text{ is found by watching an object move} \]

Measuring torque:

\[ \tau = \text{unknown torque} \]

\[ \Sigma \tau = 0 = I \alpha = I \frac{d\omega}{dt} \]

\[ FR - \tau = 0 \Rightarrow \tau = FR \]

\[ \omega \text{ is found by watching the object rotate} \]

\[ \omega = I \alpha \text{ in Nm} \]

Newton’s 3rd Law:

\[ F_{ab} = -F_{ba} \]

Two forces acting on a body as a pair:

\[ F_{an} = -F_{bn} \]

Reaction pairs are of the same kind of force. If the action is a gravitational force, then the reaction will be a gravitational force as well.

\[ F_{\text{gravity}} = \frac{G M m}{r^2} \text{ is the action} \]

\[ F_{\text{gravity}} = -\frac{G M m}{r^2} \text{ is the reaction} \]

Normal forces oppose both, one as an action, the other as a reaction.
Free Body Diagram

Every force gets an arrow. It would be best if the arrow were placed at the point where the force is applied so you can find the torque applied to it. In the event that it doesn't produce a torque, putting the arrow at the center is just as effective.

\[ \sum F_x = F_N - F_g = 0 \text{ (no motion off or through surface)} \]

\[ F_N = F_g \cos \Theta \]

\[ \sum F_y = F_y - mg \sin \Theta = ma \]

\[ a = g \sin \Theta \]

\[ F_y = mg \sin \Theta = 5 \times 9.8 \times \sin 30^\circ \]

\[ = 24.5 \text{ N} \]

\[ \Sigma F_t = T_1 - T_2 = m_2 a \Rightarrow T_2 = m_2 a + m_1 g = m_1 (a + g) \]

\[ \Sigma F_{r1} = F_{N1} - F_{y1} = 0 \]

\[ F_{N1} = m_1 g \cos \Theta \]

\[ \Sigma F_{r1} = F_{y2} - T_1 = m_2 a \]

\[ m_2 g \sin \Theta - m_1 a = T_2 = m_2 (g \sin \Theta - a) \]

\[ T_2 = T_1 = I \frac{a}{R} \]

\[ m_2 (g \sin \Theta - a) - m_1 (a + g) = I \frac{a}{R} \]

\[ \text{Total force} = m_2 g \sin \Theta - m_1 g = \frac{I (2m_2 + m_1 + m_2 a)}{I + m_1 R^2 + m_2 R^2} \]

\[ a = \frac{R^2 (m_2 \sin \Theta - m_1)}{I + m_1 R^2 + m_2 R^2} \]
Definitions

static - no motion
dynamic - motion

Equilibrium - sum of forces & torques is \( \text{ON}(m) \)

- velocity & angular velocity is constant

\[
\sum F = \text{ON}
\]
\[
F_U - F_{g1} - F_{g2} = \text{ON}
\]
\[
F_U = F_{g1} + F_{g2} = (m_1 + m_2)g
\]
\[
= (25 + 1 + 2/3)kg \times 9.8 \times \frac{9}{\sqrt{9}}
\]
\[
= 33.6 \text{ N}
\]

Unit problem

Length = [L] = [m] = [meter] = [other unit of length]
Torque = [L][M] = [Nm] = Torque

2.74

\( a = \text{constant} \)
\( t_0 = 0, \alpha = 0, x_0 = 0 \)
\( t_1 = 3s \Rightarrow x_1 = ? \)
\( t_2 = 10s \)
\( x_2 = 150m \)
\( t_3 = 20s \)

\( x_3 = x_2 + 150m \)

\( a = \frac{\Delta x}{\Delta t^2} \)

\( a = \frac{1}{2} a \times (10s)^2 \quad \frac{x_3}{x_2} = \frac{1}{2} a \times (20s)^2 \)

\( x_3 = x_2 + 150m \quad x_3 = x_2 + 150m \)

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\( \tau = I \alpha = TR \times \frac{\Delta \alpha}{\Delta t} \Rightarrow T = I \frac{\Delta \alpha}{\Delta t} \)

\( r \alpha = \omega \Rightarrow \alpha = \frac{\omega}{r} \)

\( M_y - T = M_k \quad M_y \Rightarrow M_{k+T} \)

\( M_y = a \times (M + \frac{\Delta \alpha}{\Delta t}) \)

\( a = \frac{M_y}{M + \frac{\Delta \alpha}{\Delta t}} = \frac{MR^2}{MR^2 + I} = \frac{MR^2}{MR^2 + I} \text{ positive definite} \)

\( \frac{\Delta \alpha}{\Delta t} \) can only go down.

Lit this way - it would be possible to accelerate upward.