A brother and sister are playing catch with a football.

What velocity is he throwing the ball?
What velocity is she catching the ball?
How far apart are they?

The first thing I need is a scale. I went fancy on this one. I applied a fit to the path of the football in pixels. The maximum height happens at t=.373s, y=266 px. The low point is at t=.9s, y=138px. The difference in height is 128px with an elapsed time of .527s.

\[ \Delta x = \frac{1}{2}gt^2 = \frac{1}{2}(9.81 \text{m/s}^2)(.527s)^2 = 1.36m \]

scale = 128px : 1.36m

After I have the fit and the scale, I can rescale the fit and get all of the answers I want. But that would be cheating on the use of the equations to solve the questions.

I need to find the initial velocity between the brother and sister. The distance is 982.5px or after the scaling 10.44 m. The ball is in the air for 27 frames or .9s.

\[ 982.5px \times \frac{1.36m}{128px} = 10.44m \]
\[ v_x = \frac{d}{t} = \frac{10.44m}{.9s} = 11.6 \text{m/s} \]

The ball descended from it’s maximum height 1.36m. The vertical velocity at the maximum height is 0 m/s. From that, I can find the vertical velocity when the sister catches the ball.

I can also use gravity and elapsed time find the vertical velocity as well as an idiot check.

\[ v_{fy} = \sqrt{v_0^2 + 2ad} = \sqrt{2 \times 9.81 \text{m/s}^2 \times 1.36m} = 5.17 \text{m/s} \]
\[ v_{fy} = v_0 + at = 9.81 \text{m/s}^2 \times .527s = 5.17 \text{m/s} \]
\[ v_f = \sqrt{v_{fy}^2 + v_{fx}^2} = 12.7 \text{m/s} \]

We can do the same to find the initial vertical velocity. With the scale, I find that the ball rose .681m in .373s.

\[ d = 64px \times \frac{1.36m}{128px} = .681m \]
\[ v_f^2 = v_i^2 + 2ad \]
\[ v_i^2 = v_f^2 - 2ad \]
\[ v_{iy} = \sqrt{v_f^2 - 2ad} = \sqrt{-2(9.81 \frac{m}{s^2})(-0.681m)} = 3.65 \frac{m}{s} \]
\[ v_{fy} = 0 \frac{m}{s} = v_{iy} + at \]
\[ v_{iy} = -at = -9.81 \frac{m}{s^2} \cdot 0.373s = 3.65 \frac{m}{s} \]
\[ v_i = \sqrt{(3.65 \frac{m}{s})^2 + (11.6 \frac{m}{s})^2} = 12.16 \frac{m}{s} \]

I can do a further idiot test by showing that I can get from \( t=0 \)s to \( t=0.9 \)s in position and velocity. I'm going to do the horizontal because that is trivially true. The ball goes up .682m and then it comes down 1.36m or down .682m over .9s.

\[ y = v_0t + \frac{1}{2}at^2 \]
\[ y = 3.65 \frac{m}{s} \cdot 0.9s - \frac{1}{2} \cdot 9.81 \frac{m}{s^2} \cdot (0.9s)^2 = -0.682m \]
\[ v_{fy} = 3.65 \frac{m}{s} - 9.81 \frac{m}{s^2} \cdot 0.9s = -5.17 \frac{m}{s} \]
\[ v_{fy} = \sqrt{(3.65 \frac{m}{s})^2 + 2 \cdot 9.81 \frac{m}{s^2} \cdot (0.682m)} = 5.17 \frac{m}{s} \]

I can also look for the angular velocity of the football. We'll use the laces to define the angular position of the football as the white sticks out on the brown ball. We'll also define laces to the boy’s right, or towards the camera as 0. In the first frame, the football laces are down or \(-90^\circ\). It rotates 6 times and then ends with the laces away from the camera or \(180^\circ\). That totals to 6.75 rotations in .9s. \( \omega = \frac{\Delta \theta}{\Delta t} = \frac{6.75 \text{rev}}{0.9s} = 7.5 \text{rev/s} \)

\[ \omega = 7.5 \frac{\text{rev}}{s} \cdot 2 \pi \frac{\text{rad}}{\text{rev}} = 47.1 \frac{\text{rad}}{s} \]