Rotational Work + Power

Linear Work

Angular Work

Rotational Work

Power

\[ P = Fv \]

\[ P = T \omega \]

\[ W = Fd \]

\[ W = \tau \theta \]

Angular Momentum

\[ L = r \times p = \vec{m} \times \vec{v} \] definition

\[ \dot{L} = \dot{mv} = m \dot{v} \] more useful to this class

\[ L = \vec{mv} = 90 \text{ Nm} \cdot \text{m} = 90 \text{ Nm} \cdot \text{m/s} 

\[ \dot{L} = \dot{mv} = 27 \text{ Nm} \cdot \text{s} \]

\[ L = I \omega = mr^2 / \omega = mrv \]

Angular momentum is often conserved

If there is no torque, then Angular momentum is conserved

If something is moving in a circle, think about chaing the Angular momentum conservation instead of momentum.

Ice skater spinning

\[ I_i = \frac{1}{2} M R^2 \]

\[ = \frac{1}{2} \times 70 \text{ kg} \times (0.25 \text{ m})^2 \]

\[ = 2.5 \text{ kg m}^2 \]

\[ I_f = 5 \text{ kg m}^2 \]

\[ \frac{5 \text{ kg m}^2 \times 2.5 \text{ m/s}^2}{2 \times 5 \text{ kg m}^2} = 4 \text{ m/s} \]

Child "collides" with merry-go-round. What is final velocity?

\[ v = \sqrt{\frac{2 \times \text{ work}}{\text{momentum}}} \]

\[ m = \frac{5 \text{ kg}}{20 \text{ kg}} \]

\[ a = \frac{20 \times 5 \text{ m/s}^2}{630 \text{ kg m/s}^2} = 0.76 \text{ m/s}^2 \]
Kepler's 3rd Law: $T^2 \propto r^3$ from $\frac{m\nu}{r} = G\frac{Mm}{r^2} + \nu = \frac{2\pi}{T}$

Kepler's 2nd Law: $\frac{1}{2} L = \text{Conserved}$

It can be shown that

$E = \text{Conserved}$ $L = I\omega$ $KE = \frac{1}{2} I\omega^2 = \frac{\omega^2}{2\ell}$

$E = PE + KE_{rad} + KE_{tan}$

$= -G\frac{Ma}{r} + \frac{1}{2} m\nu_{rad}^2 + \frac{1}{2} \ell\nu_{tan}^2$

$= G\frac{Ma}{r} + \frac{1}{2} \ell\nu_{rad}^2 + \frac{C^2}{2r^2}$

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The math is easy to do this.

Finding the angle is hard.

The new radial position and velocity can be easily found.

This is how mass does orbits.

Inertial forces are not.

It is a correct understanding, but isn't useful.

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Ladder problem

$\Sigma F_x = N_x + W - F_x = 0$\n\n$N_x = W_1 - W_2 = 1500\text{ N}$

$\Sigma F_y = N_y - W = 0$

$N_y = 4000\text{ N}$

$N = N_x = N_y = 4000\text{ N}$

$\Sigma F_x = \nu_x + F_x = \nu_y + F_y = 0$

$\nu = \frac{1500}{4980} \approx 0.3\text{ m/s}$

$\nu = \frac{1500}{9500} \approx 0.15\text{ m/s}$

$\nu = \frac{1500}{9500} \approx 0.15\text{ m/s}$

If $\nu$ is the minimum needed to hold the ladder, it suggests something bigger.
Force of Wiper Arm:

\[ T = \frac{525.5 \text{ kN} \cdot \text{m} \cdot \sin 15^\circ}{9 \text{ m} \cdot \sin 15^\circ} = 48.2 \text{ kN} \]

\[ \Sigma F_x = F_{HA} - T_x = 0 \]
\[ F_{HA} = T \cos 5^\circ \]
\[ = 48.2 \text{ kN} \cos 5^\circ \]
\[ = 48.0 \text{ kN} \]
\[ F_y = (48.0 \text{ kN} \cdot 72.8 \text{ mm}) = 972 \text{ mm} @ 5^\circ \]

\[ \Sigma F_y = F_{HA} - T_y - W = 0 \]
\[ F_{HA} = W + T_y \]
\[ = 7 \text{ kN} + 9.8 \text{ kN} \sin 5^\circ \]
\[ = 7 \text{ kN} + 9.8 \text{ kN} \sin 5^\circ \]
\[ = 7 \text{ kN} + 9.8 \text{ kN} \sin 5^\circ \]

\[ \Sigma \alpha = \frac{1}{2} C_x = C_x = 525.5 \text{ kN} \cdot \text{m} \]
\[ I = \frac{1}{12} I_y \cdot \text{m}^2 = \frac{1}{12} \times 30^2 \text{ m}^4 = 937.5 \text{ m}^4 \]
\[ \alpha = \frac{525.5 \text{ kN} \cdot \text{m}}{937.5 \text{ m}^4} = 0.567 \text{ rad} \]

\[ E = E_k \]
\[ \rho C = \frac{1}{2} \rho v^2 \cdot \text{m} \cdot \text{m} \cdot \text{m} \cdot \text{sec} \]
\[ m \rho v^2 = \frac{1}{2} \rho v^2 \]
\[ h = \frac{v^2}{2g} \sin 46^\circ \]
\[ = 10 \text{ m/s} \sin 46^\circ = 6.4 \text{ m} \]
\[ \omega = \sqrt{\frac{2mgh}{I}} = \frac{2 \cdot 7 \text{ kN} \cdot 9.8 \text{ m/s}^2 \cdot 6.4 \text{ m}}{437.5 \text{ m}^4} = 972 \text{ rad/s} \]