Chapter 35, example problems:

(35.02) Two radio antennas A and B radiate in phase. B is 120 m to the right of A. Point Q along extension of line AB. 40 m to the right of B. Frequency and wavelength can be varied.

(a) Destructive interference at Q. Longest wavelength?

\[ AQ - BQ = \frac{1}{2}\lambda \Rightarrow 120 \text{ m} = \frac{1}{2}\lambda \Rightarrow \lambda = 240 \text{ m}. \]

(Note: If you change \(\frac{1}{2}\) to \(\frac{3}{2}\), \(\frac{5}{2}\), etc., you will get other possible shorter wavelengths. But the problem asks for the longest wavelength.)

(b) Constructive interference at Q. Longest wavelength?

Since \( AQ - BQ \) can not be zero, we demand \( AQ - BQ = \lambda \).

\[ \Rightarrow 120 \text{ m} = \lambda \Rightarrow \lambda = 120 \text{ m}. \]

(35.16) Coherent light. Frequency \(6.32 \times 10^{14} \text{ Hz}\). Pass through two thin slits. Fall on a screen 85.0 cm away. 3rd bright fringe occurs at \(\pm 3.11 \text{ cm}\) on either side of center bright fringe.

(a) Slit separation \(d\).

\[ d \sin \theta = m\lambda = 3 \times (3 \times 10^8 \text{ m/s} / 6.32 \times 10^{14} \text{ Hz}) \]

\[ \tan \theta = 3.11 \text{ cm} / 85 \text{ cm}. \]

\[ \therefore d = 3 \times (3 \times 10^8 \text{ m/s} / 6.32 \times 10^{14} \text{ Hz}) / (3.11 \text{ cm} / 85 \text{ cm}) = 3.89 \times 10^{-5} \text{ m} \]

or \[ d = 3.89 \times 10^{-3} \text{ cm}. \]

(Note: The unit Hz is just \(s^{-1}\).)

(b) Location of the third dark fringe? The first dark fringe occurs midway between the center (zeroth) bright fringe and the first bright fringe. So the third dark fringe occurs midway between the second and third bright fringes. Hence we solve \( d\theta = 2.5 \times (3 \times 10^8 \text{ m/s} / 6.32 \times 10^{14} \text{ Hz}) \) to get \( \theta = 0.0305\). Multiplying it by 85 cm gives 0.0259 m, or \(0.0259 \text{ cm}\) being the distance on the screen between the third dark fringe and the center of the center bright fringe.

(35.26) Two antennas, 9.00 m apart. Radiate in Phase at 120 MHz. Receiver, placed 150 m from both antennas measured an intensity \(I_0\). Receiver moved so that it is now 1.8 m closer to one antenna than to the other.

(a) phase difference:

\[ \phi = 2\pi \times 1.8 \text{ m} / \lambda = 2\pi \times 1.8 \text{ m} / (3.00 \times 10^8 \text{ m/s} / 120 \times 10^6 \text{ Hz}) = 4.524 \text{ rad}. \]

(b) Intensity at new position:

\[ I = I_0 \cos^2 \left( \frac{\phi}{2} \right) = 0.4064 I_0. \]

Why \( I = I_0 \cos^2 \left( \frac{\phi}{2} \right)\)?

Recall the phasor diagram at right:
Glass plate, 9.00 cm long, placed in contact with a second plate. Held at a small angle by a metal strip 0.0800 mm thick under one end. Air between the plates. Illuminated from above by light with wavelength \( \lambda = 656 \text{ nm} \). Number of interference fringes per centimeter?

The interference is between light reflected from the bottom surface of the top plate, and the top surface of the bottom plate. Other surfaces are not relevant. (If you want to know why it is so, come to class and listen to the lectures.) The left end is the center of a dark fringe, because the lower reflection gets a 180° phase shift — due to reflection by a denser medium, but not the upper Reflection, which is a reflection by a lighter medium. The difference in path lengths between the two paths is zero on the left edge, and it increases continuously to the maximum value of \( 2 \times 0.0800 \text{ mm} = 0.1600 \text{ mm} \) on the right edge for light coming down essentially perpendicular to the plates (called normal incidence). The phase difference between the two paths on the right edge of the plates is \( 2\pi \times \frac{\text{path length difference}}{\lambda} = 2\pi \times \frac{0.1600 \text{ mm} / 656 \text{ nm}}{574.68 \text{ rad.}} \), but for this problem, it is only necessary to look at the part \( \frac{0.1600 \text{ mm} / 656 \text{ nm}}{243.9} \), which is slightly below 244. It means that there are almost 244 wavelengths in this maximum path-length difference. Thus the center of the 244th dark fringe besides the one on the left edge of the plates is just outside the right edge of the plates and therefore can not be seen. But the bright fringe just to the left of this not-fully-observable dark fringe needs only a path-length difference of 243.5 wavelengths, and is therefore already fully developed. Since there is a bright fringe to the left of each dark fringe, including the first one, it is clear that the total number of bright fringes that can be seen is 244. Dividing it by 9.00 cm gives **27.11 bright fringes per centimeter** that can be seen.

(Note that the inverse of 27.11 number per centimeter is 0.0369 cm. This is the spacing between the neighboring bright fringes that can be seen from above.)

Michelson interferometer. Jan uses \( \lambda_1 = 606 \text{ nm} \). He moved the movable mirror away from him by some distance, and counted 818 bright fringes. Then Linda changes to \( \lambda_2 = 502 \text{ nm} \), and moved the movable mirror toward her by some distance, and counted also 818 bright fringes. The fringes moved in opposite directions across the line. Find the distances moved by the movable mirror in each case.

(a) Jan moved the mirror \( 818 \times 606 \text{ nm} / 2 = 2.479 \times 10^{-4} \text{ m} \) away from him, and Linda moved the mirror \( 818 \times 502 \text{ nm} / 2 = 2.053 \times 10^{-4} \text{ m} \) toward her. (Note: Light travels to the movable mirror and back, so the distance moved by the mirror should be counted twice.)

(b) The resultant displacement of the mirror = \( 2.479 \times 10^{-4} \text{ m} - 2.053 \times 10^{-4} \text{ m} = 0.426 \times 10^{-4} \text{ m} \), or \( 4.26 \times 10^{-5} \text{ m} \).
Two radio antennas, 200 m apart, at A and B, radiate in phase. Frequency 5.80 MHz. Receiver at C. BC is perpendicular to AB. Find location of C for destructive interference.

We need \( \overrightarrow{AC} - \overrightarrow{BC} = (1/2) \lambda \). ⇒ 
\[
\sqrt{x^2 + (200 \text{ m})^2} - x = (1/2) \left( 3 \times 10^8 \text{ m/s} / 5.80 \times 10^6 \text{ Hz} \right) = 25.86 \text{ m}, \text{ or,}
\]
after adding x to both sides and then squaring both sides, we obtain:
\[
x^2 + (200 \text{ m})^2 = x^2 + 25.86 \text{ m} \times x + (25.86 \text{ m})^2. \text{ After simplifying, we obtain}
\]
25.86 m \( \times x = 39331 \text{ m}^2 \), giving \( x = 1521 \text{ m} \).

Two-slit interference. The two slits are of different widths. Distant screen. Amplitude from the first slit = \( E \). Amplitude from the second slit = \( 2E \).

(a) \( I(\phi) = ? \)
The intensity is proportional to the net amplitude squared. A phasor diagram tells you how to find the net amplitude. For \( \phi = 0 \), the net amplitude is \( 3E \). For \( \phi \neq 0 \), the net amplitude follows the law of cosine (as suggested by the diagram):
\[
\sqrt{(2E)^2 + (E)^2 - 2(2E)(E) \cos (180^\circ - \phi)} = \sqrt{5E^2 + 4E^2 \cos \phi}.
\]
The ratio of the two net amplitudes is \( \sqrt{(5/9) + (4/9) \cos \phi} \). Therefore, \( I(\phi) = I_0 \left[ (5/9) + (4/9) \cos \phi \right] \), where \( I_0 \) is just \( I(\phi) \) at \( \phi = 0 \).
(Note: To get amplitude \( 2E \) from one slit and \( E \) from the other slit, all one needs to do is to make the first narrow slit twice as wide as the second narrow slit.)

(b) Graph of \( I(\phi) = I_0 \left[ (5/9) + (4/9) \cos \phi \right] \) (for \( \phi \) between \(-5\pi \) and \(+5\pi \)):
(35.51) Red light, wavelength 700 nm, passes through two slits. Monochromatic visible light of another wavelength also passes through the two slits. Center of the third bright fringe \((m = 3)\) of the red light appears red, with no mixture of the other color. Possible wavelength for the second visible light? We use the formula “\(d \sin \theta = 3 \lambda_1\)” for the red light, and “\(d \sin \theta = (m + 1/2) \lambda_2\)” for the second visible light. Clearly, it is the same \(d\) in both formulas (same two slits), and it is the same \(\theta\) in both formulas (same spot on the screen). Thus we must have \(3 \times 700 \text{ nm} = (m + 1/2) \lambda_2\). Taking \(m = 0\) gives \(\lambda_2 = 1400 \text{ nm} \) — not visible, discard. Taking \(m = 1\) gives \(\lambda_2 = 466.7 \text{ nm} \) — visible, accept. Taking \(m = 2\) gives \(\lambda_2 = 280.0 \text{ nm} \) — not visible, discard. There is clearly no need to try larger \(m\). So the second visible light must have the wavelength 466.7 nm. It is clear that we do not need to know \(d\), because we can eliminate it between the two equations. (To do this problem, you need to know that visible light has wavelength between 400 nm and 700 nm.)

(35.54) White light reflects at normal incidence from top and bottom surface of a glass plate \((n = 1.52)\). Air above and below. Constructive interference observed for \(\lambda_1 = 477.0 \text{ nm} \) (in air). Next longer wavelength for constructive interference is \(\lambda_2 = 540.6 \text{ nm} \). Plate thickness \(t = ?\)

We must have: \(2t = (m_1+1/2) \lambda_1 = (m_2+1/2) \lambda_2\), where \(m_1\) and \(m_2\) are integers. (The \(1/2\) part is needed because the reflection from the top surface is from a denser medium, which gets a \(\pi\) phase shift, but not that from the bottom surface. Here \(\lambda_1\) and \(\lambda_2\) are both those in the medium with \(n = 1.52\). So using the wavelengths in air the equation changes to \(2nt = (m_1+1/2) \lambda_1 = (m_2+1/2) \lambda_2\). Thus the ratio of \(\lambda_1\) and \(\lambda_2\) should be the ratio of two half odd integers. Let us first evaluate this ratio: \(\lambda_1 / \lambda_2 = 477 / 540.6 = 0.8824 = 15 / 17 = 7.5 / 8.5\). Thus \(m_2\) must be 7 and \(m_1\) must be 8. We therefore find \(t = (m_1+1/2) \lambda_1 / 2n = 8.5 \times 477.0 \text{ nm} / 2 \times 1.52 = 1334 \text{ nm} = 1.334 \mu\text{m}\). Of course, you can also obtain the same answer using \(t = (m_2+1/2) \lambda_2 / 2n\).

(35.58) An oil tanker spills a larger amount of oil \((n = 1.45)\) into the sea \((n = 1.33)\).

(a) Looking down from overhead. Predominant wavelength of light seen at where oil thickness \(= 380 \text{ nm}\)? In this case, the reflection from the top surface of oil film gets a \(\pi\) phase shift, but not that from the bottom surface of the oil film. So to get the dominant color seen, we apply \(2t = (m + 1/2) \lambda / n\), and try all integers for \(m\) until a visible wavelength is found. For \(m = 0\), we get \(\lambda_0 = 2204 \text{ nm} \) — too long. For \(m = 1\), we get \(\lambda_1 = 2204 \text{ nm} / 3 = 734.7 \text{ nm} \) — too long. For \(m = 2\), we get \(\lambda_2 = 2204 \text{ nm} / 5 = 440.8 \text{ nm} \) — visible, accept. For \(m = 3\), we get \(\lambda_3 = 2204 \text{ nm} / 7 = 314.9 \text{ nm} \) — too short. We clearly do not need to try any larger \(m\). Thus, the predominant visible wavelength is 440.8 nm. Its color is blue (c.f., Table 32.1).
(b) For the predominant transmitted-light wavelength, we need destructive interference for the reflected light so no reflection occurs. Thus we solve $2t = m \lambda / n$, and again try all integer $m$. For $m = 1$, we get $\lambda_1 = 1102 \text{ nm} — \text{too long.}$
For $m = 2$, we get $\lambda_2 = 1102 \text{ nm} / 2 = 551 \text{ nm} — \text{visible, accept.}$ For $m = 3$, we get $\lambda_3 = 1102 \text{ nm} / 3 = 367.3 \text{ nm} — \text{too short.}$ Thus the predominant transmitted visible light has $\lambda = 551 \text{ nm (in air).}$ The color is green (c.f., Table 32.1).