Chapter 33, example problems:

(33.10) (a) Since the two interface normals are parallel to each other, we must have $\theta_b = \theta_c$. We also have

$$n_a \sin \theta_a = n_g \sin \theta_b,$$

and

$$n_g \sin \theta_c = n_m \sin \theta_d.$$ So

$$\sin \theta_d = (n_g / n_m) \sin \theta_c = (n_g / n_m) (n_a / n_g) \sin \theta_a = (n_a / n_m) \sin \theta_a = (1.000 / 1.329) \sin 41.3^\circ = 0.4966,$$

which gives $\theta_d = 29.78^\circ$.

(b) Methanol changed to an unknown liquid. $\theta_d$ is now given to be $20.2^\circ$. Then

$$n_m = n_a \sin \theta_a / \sin \theta_d = 1.000 \sin 41.3^\circ / \sin 20.2^\circ = 1.911.$$

(33.16) $60^\circ - 15^\circ = 45^\circ$.

$1.33 \sin 45^\circ = 1.52 \sin \theta_g$, giving $\theta_g = 38.2^\circ$.

But $\theta_g$ is now measured with respect to the tilted normal. So relative to the vertical the angle in glass is $38.2^\circ + 15^\circ = 53.2^\circ$.

(See figure.)

(33.22) (a) $1.52 \sin \theta = 1.00 \sin 90^\circ$ gives

$$\sin \theta = 1/1.52 = 0.6579,$$

or $\theta = 41.14^\circ$.

This is the largest $\alpha$, because if $\alpha$ is larger than this angle, then $\theta$ is less than $41.14^\circ$, and light will be able to come out of the face AC at an angle less than $90^\circ$ with the surface normal there.

(b) Change $1.00$ to $1.333$, and we get the largest $\alpha$ to be $90^\circ - 61.28^\circ = 28.72^\circ$.

(33.26) $1.333 \sin 53.0^\circ = n \sin (90^\circ - 53.0^\circ) = n \cos 53.0^\circ$,

giving: $n = 1.333 \tan 53.0^\circ = 1.769$.

Note that this is exactly the situation when the reflected light is 100% polarized in the direction perpendicular
to the scattering plane. That is, this special incident angle is called Brewster’s angle or polarizing angle for the given $n$ values on the two sides of the interface.

(33.34) If the incoming light is unpolarized and has intensity is $I_0$, then after going through the first polarizing filter, the light becomes polarized vertically, and its intensity becomes $I_0/2$. After going through the second filter, its polarization is projected to $23^\circ$ to the vertical, and its intensity becomes $(1/2) I_0 \cos^2 23^\circ$. Then after going through the third filter, its polarization direction gets projected again, to $62^\circ$ from vertical, so light comes out with its polarization at $62^\circ$ from vertical, and its intensity is now:

$$I = (1/2) I_0 \cos^2 23^\circ \cdot \cos (62^\circ - 23^\circ) = \frac{0.5 \times 0.847 \times 0.604}{2} I_0 = 0.256 I_0.$$ 

Note that each projection factor for the intensity (which is proportional to the amplitude squared) is cosine square of the angular change of the polarization direction. So the projection factor associated with the third filter involves the angle $62^\circ - 23^\circ$.

Now the problem says that $0.256 I_0$ is equal to 75 W/cm$^2$, so $I_0$ must be $293.1$ W/cm$^2$. If the second filter is now removed, the new reduction factor of intensity becomes: $(0.5) \times \cos^2 62^\circ = 0.5 \times 0.220 = 0.110$. Therefore the new out-coming intensity is: $293 \text{ W/cm}^2 \times 0.110 = 32.3 \text{ W/cm}^2$.

(33.40) $\lambda = 490$ nm in air. From laser to photocell is 17.0 ns. Slab of glass 0.840 m thick is inserted perpendicular to the light beam. It now takes light 21.2 ns to reach the photocell. $\lambda_{\text{glass}} = ?$

The essential concept to solve this problem is that light travels slower in the glass than in air (or vacuum), by a factor of $n$, and so the wavelength in the glass is shorter than it is in air since the frequency of light does not change as light enters into the glass.

Extra travel time due to the insertion of the slab of glass in its path $= 21.2 \text{ ns} - 17.0 \text{ ns} = 4.2 \text{ ns}$. It must be equal to $0.840 \text{ m} / v_{\text{glass}} - 0.840 \text{ m} / c$.

So $0.840 \text{ m} / v_{\text{glass}} = 4.2 \text{ ns} + 0.840 \text{ m} / 3 \times 10^8 \text{ m/s} = 7.0 \text{ ns}$. Hence $v_{\text{glass}} = 0.840 \text{ m} / 7.0 \text{ ns} = 1.20 \times 10^8 \text{ m/s}$. Since frequency in glass = frequency in air, we have $v_{\text{glass}} / c = \lambda_{\text{glass}} / \lambda_{\text{air}}$. Hence $\lambda_{\text{glass}} = \lambda_{\text{air}} \times \left( \frac{1.20 \times 10^8 \text{ m/s}}{3 \times 10^8 \text{ m/s}} \right) = 196 \text{ nm}$.

(33.44) Glass plate, 2.50 mm thick. Index of refraction $n = 1.40$, placed between a point source of light with wavelength 540 nm and a screen. Source to screen: 1.80 cm. Here the main concept needed to solve this problem is that light wavelength becomes shorter in a medium with an index of refraction $n > 1$, so that more
wavelengths can fit into the thickness of glass. It should be added to the number of wavelengths which can be fitted into the remaining part of the light path to get the total number of wavelengths. So the number of wavelengths
\[
= 2.5 \text{ mm}/(540 \text{ nm} / 1.40) + (1.80 \text{ cm} – 2.50 \text{ mm})/540 \text{ nm}
\]
\[
= 6481.5 + 28703.7 = 35185.2
\]
(This answer has no unit!)

\[
(33.46) \quad 1.0 \sin \alpha = n \sin \beta
\]
\[
\sin \alpha = 1.5/\sqrt{1.2^2 + 1.5^2}
\]
giving \( \sin \beta = 0.58580 \)
But \( \sin \beta = x/\sqrt{x^2 + 4.0^2} \)
So \( x^2 = 0.34316 \times 16.0 \)
or \( x = (8.3590)^{1/2} = 2.891 \text{ m} \).

Adding it to 1.5 m giving 4.391 m as the distance between the key and the edge of the pool at the bottom of the pool.

\[
(33.52) \quad \theta = 90^\circ - 30^\circ = 60^\circ.
\]
It needs to be the critical angle \( \theta_c \) for total reflection at P if \( n \) is the maximum value for such a purpose.
So \( 1.62 \sin 60^\circ = n_{\text{max}} \sin 90^\circ \),
giving \( n_{\text{max}} = 1.62 \times \sqrt{3}/2 = 1.403 \).
Why is this the maximum \( n \) because the left hand side of the equation is fixed, so if \( n \) is larger than this value it would have to be multiplied by a smaller value than \( \sin 90^\circ \), which means that the refracted angle will be less than \( 90^\circ \). Light then would not be totally reflected at P. Note that this \( n_{\text{max}} \) is still less than 1.62. This should be clear, since once \( n \) reaches 1.62, the interface would have disappeared, and total reflection would certainly not happen at this non-existant interface. Hence this \( n_{\text{max}} \) has to be less than 1.62.