Chapter 19, two more example problems:

(19.58) Certain ideal gas. \( C_V \) given. Initially, \( p_0, V_0, T_0 \). Expands isobarically to \( 2V_0 \).
Then expands adiabatically to \( 4V_0 \).
(a) Draw the processes in a \( pV \) diagram:
(b) \( W_{total} = W_{a→b} + W_{b→c} \).
\[ W_{a→b} = p_0 \times (2V_0 - V_0) = p_0 V_0. \]
\[ W_{b→c} = (p_0 \times 2V_0 - p_c \times 4V_0) / (\gamma - 1) \]
[Eq. (19.26) in text]
To find \( p_c \), we use \( p_b V_b^\gamma = p_c V_c^\gamma \),
or, \( p_0 (2V_0)^\gamma = p_c (4V_0)^\gamma \), which gives
\[ p_c = p_0 (1/2)^\gamma. \]
Hence
\[ W_{b→c} = (2p_0 V_0 - 4(1/2)^\gamma p_0 V_0) / (\gamma - 1) \]
\[ = 2 \left[ (1 - 2^{1-\gamma}) / (\gamma - 1) \right] p_0 V_0. \]
[Note that \( 1-\gamma \) is actually negative, so \( 2^{1-\gamma} \) is actually less than unity.]
Alternatively,
\[ W_{b→c} = \int_{V_b}^{V_c} pVdV = p_b V_b^\gamma \int_{V_b}^{V_c} V^{-\gamma+1}dV \]
\[ = p_b V_b^\gamma \left( V_c^{1-\gamma+1} - V_b^{1-\gamma+1} \right) / (\gamma - 1) \]
\[ = p_0 (2V_0)^\gamma \left( (4V_0)^{1-\gamma+1} - (2V_0)^{1-\gamma+1} \right) / (\gamma - 1) \]
\[ = 2 \left[ (1 - 2^{1-\gamma}) / (\gamma - 1) \right] p_0 V_0. \]
That is, we get the same result.

(c) \( T_{final} = T_c \). But \( T_b V_b^{\gamma-1} = T_c V_c^{\gamma-1} \), or \( T_c = T_b (1/2)^{\gamma-1} = 2T_0 (1/2)^{\gamma-1} \)
\[ = (4/2^\gamma) T_0. \]
Hence \( T_{final} = (4/2^\gamma) T_0. \)

(d) \( Q_{total} = Q_{a→b} + Q_{b→c} \). But \( Q_{b→c} = 0 \), and
\[ Q_{a→b} = n C_p (T_b - T_a) = n (C_V + R) (2T_0 - T_0) = n (C_V + R) T_0 \].
Hence
\[ Q_{total} = n (C_V + R) T_0. \]
This \( Q \) is positive, so it is a heat input. (Positive \( Q \) always means heat input.)

(19.60) Turbochargers and Intercoolers.

(Adiabatic compression)
\[
\begin{align*}
p_a &= 1.01 \times 10^5 \text{ Pa} \\
\rho_a &= 1.23 \text{ kg/m}^3 \\
T_a &= 15.0 \text{ °C}
\end{align*}
\]
\[
\begin{align*}
p_b &= 1.45 \times 10^5 \text{ Pa} \\
\rho_b &= ? \text{ kg/m}^3 \\
T_b &= ? \text{ °C}
\end{align*}
\]

(Isobaric)
\[
\begin{align*}
p_c &= 1.45 \times 10^5 \text{ Pa} \\
\rho_c &= ? \text{ kg/m}^3 \\
T_c &= 15.0 \text{ °C}
\end{align*}
\]
(a) Draw the two processes in a \( pV \) diagram:
(b) \( V = 575 \text{ cm}^3 \) for one of the engine cylinders.
Find \( m \) of air in the intercooler at \( p_c = 1.45 \times 10^5 \text{ Pa} \) to fill this \( V \). We need \( \rho_c \), the
air density in the state \( c \), since \( m = \rho_c V \). But
we need \( \rho_b \) first: Using
\[ p_a V_a^{\gamma} = p_b V_b^{\gamma} \], and \( \rho_a V_a = m = \rho_b V_b \),
we obtain \( \rho_a / \rho_a^{\gamma} = p_b / p_b^{\gamma} \). Hence
\[ \rho_b = \rho_a (p_b / p_a)^{\gamma} \]
\[ = 1.23 \text{ kg/m}^3 \times \]
\[ (1.45 \times 10^5 \text{ Pa} / 1.01 \times 10^5 \text{ Pa})^{1/1.40} \]}
= 1.5925 kg/m³. [For air, which contains diatomic molecules, \( \gamma = 7/5 = 1.40 \).]

The air that is forced into the cylinder is at the end of the processes. That is, it is in the state \( c \). Thus the mass \( m \) to be forced into an engine cylinder is equal to \( \rho_c V \). To find \( \rho_c \), we first find \( T_b \), using \( T_a \gamma / p_a \gamma^{-1} = T_b \gamma / p_b \gamma^{-1} \), to obtain

\[
T_b = T_a \frac{p_b}{p_a} (\gamma^{-1}) / \gamma = 288.0 \text{ K} \times (1.45 \times 10^5 \text{ Pa} / 1.01 \times 10^5 \text{ Pa})^{0.40/1.40} = 319.35 \text{ K}.
\]

Then we apply the formula \( \rho = \rho M / RT \) to the process \( b \rightarrow c \) to get

\[
\rho_b T_b = \rho_c T_c,
\]

or \( \rho_c = \rho_b (T_b / T_c) = 1.5925 \text{ kg/m}^3 \times (319.35 \text{ K} / 288.0 \text{ K}) = 1.766 \text{ kg/m}^3 \). Thus

\[
m = 1.766 \text{ kg/m}^3 \times 575 \times 10^{-6} \text{ m}^3 = 1.015 \times 10^{-3} \text{ kg}.
\]

If we do not use the turbocharger and intercooler, we would be filling the cylinder with air at 1.01 \( \times 10^5 \text{ Pa} \), for which the density is given to be 1.23 kg/m³, so the air filling the cylinder would have the mass \( m' = 1.23 \text{ kg/m}^3 \times 575 \times 10^{-6} \text{ m}^3 = 7.0725 \times 10^{-4} \text{ kg} \). Hence the mass ratio \( m/m' = 1.436 \).

According to the first sentence of the problem, this mass ratio also gives the percentage increase in power obtained by using the turbocharger and intercooler. (Here power refers to \( Q_H \) gained by the engine per cycle through chemical reaction of gasoline and oxygen in the air that is forced into the cylinder. Here the amount of oxygen in the cylinder is the key factor, since we can always put in the right amount of gasoline.) Hence the percentage increase in power is equal to \( 43.6 \% \).

(c) If intercooler not used, we would have to use \( \rho_b \) rather than \( \rho_c \) to calculate \( m \).

That is, we would now have

\[
m = \rho_b V = 1.5925 \text{ kg/m}^3 \times 575 \times 10^{-6} \text{ m}^3 = 9.157 \times 10^{-4} \text{ kg}.
\]

Hence we would get for the mass ratio \( m/m' = 9.157 \times 10^{-4} \text{ kg} / 7.0725 \times 10^{-3} \text{ kg} = 1.295 \). So the percentage gain in power per cycle is now substantially smaller.

The basic concept of this problem is the following: The density of air is too low so we cannot put much air into the cylinder of a fixed volume (per cycle). We would like to increase the air we can put into the cylinder. Then we can also put in more gasoline per cycle, and then we can get more power per cycle through the reaction of the gasoline with the oxygen in the air. This is done by first adiabatically compressing the air to increase its density. But it makes the air hotter than before the compression, so we can cool it back to the original temperature of air at a constant pressure, which will increase the air density a bit more. Why not compress the air isothermally in the first place so we don’t have to cool it afterwards? Because adiabatic process can be done rapidly whereas isothermal process must be slow. This is because heat must flow out in the latter process, so it must be slow. Isobaric cooling is also much better than isochoric cooling, since if volume is held fixed for a fixed amount of air, its density will not change, whereas for isobaric cooling the density will increase further, since its volume is reduced as temperature drops (from the ideal gas law). Unfortunately, this process is somewhat slow, because you have to wait until the air cools down. Thus we normally do not compress the air by a large factor, which would raise the temperature of the air by too much, which would then take a long time to cool down. You would then have to not wait until the air cools down.
Chapter 20, six example problems:

(20.04) Gasoline engine. Power output (or rate of work output) = 180 kW (about 241 hp), which just means \( \frac{dW}{dt} = 180 \text{ kW} \). Thermal efficiency \( e = 28.0\% \).

(a) \( \frac{dQ_H}{dt} = ? \) (Rate of heat input.) \( Q_H \) is positive, being a heat input.
   We have \( e = \frac{W}{Q_H} = \frac{\left( \frac{dW}{dt} \right)}{\left( \frac{dQ_H}{dt} \right)} \).
   \( \therefore \frac{dQ_H}{dt} = \frac{\left( \frac{dW}{dt} \right)}{e} = \frac{180 \text{ kW}}{0.28} = 643 \text{ kW} \).

(b) \( \frac{d|Q_C|}{dt} = ? \) (Rate of heat rejected to the cold reservoir, i.e., the environment)
   Since \( Q = \Delta U + W \), but for a cycle, \( \Delta U = 0 \), and \( Q_{cycle} = Q_H + Q_C \), \( W_{cycle} = W \), we have \( Q_H + Q_C = Q_H - |Q_C| = W \). \( Q_C \) is negative, being a heat output.
   Hence \( \frac{d|Q_C|}{dt} = \frac{dQ_H}{dt} - \frac{dW}{dt} = 643 \text{ kW} - 180 \text{ kW} = 463 \text{ kW} \).

(20.08) Otto cycle. Compression ratio = 8.8.

(a) Otto cycle is made of two adiabatic and two isochoric processes:
   \( Q_H = Q_{d \rightarrow a} = nC_V(T_a - T_d) \),
   \( Q_C = Q_{b \rightarrow c} = nC_V(T_c - T_b) \),
   (because \( W = 0 \) for isochoric processes).
   \( T_a > T_d \Rightarrow Q_H > 0 \), as it should be.
   \( T_c < T_b \Rightarrow Q_C < 0 \), as it should be.
   \( |Q_C| = nC_V(T_b - T_c) \).
   \( e = 1 - |Q_C| / |Q_H| \)
   \( = 1 - (T_b - T_c) / (T_a - T_d) \) here.
   But \( T_aV_a^{-1} = T_bV_b^{-1} \), and
   \( T_cV_c^{-1} = T_dV_d^{-1} \).
   So \( e = 1 - \left( \frac{V_a/V_b}{} \right)^{-1}T_a - \left( \frac{V_d/V_c}{} \right)^{-1}T_d \).
   Now using \( V_d = V_a \) and \( V_c = V_b \), we find
   \( e_{ideal} = 1 - (V_a / V_b)^{-1} = 1 - \left( \frac{1}{8.8} \right)^{140-1} = 0.581 = 58.1\% \).
   [Note that we have derived the formula for \( e \) here, but the formula can actually be found in the formula sheet.]

(c) If the compression ratio is changed to 9.6, \( e_{ideal} \) would changed to
   \( 1 - \left( \frac{1}{9.6} \right)^{140-1} = 0.595 = 59.5\% \).
   The increase in the ideal efficiency is \( 59.5\% - 58.1\% = 1.4\% \).

(20.10) Room air conditioner. Coefficient of performance \( K = 2.90 \) on a hot day, and uses 850 W of electric power.

(a) How many Joules of heat are removed from the room per minute?
   The directions of energy flows shown in the figure at right is the key to solve this problem. The heat removed from the room is \( |Q_C| \), whereas the heat \( |Q_H| \) is dumped to the hot air outside of the house, and it is equal to \( |Q_C| + |W| \), whereas the coefficient of performance \( K \) is equal to \( |Q_C| / |W| \) (so
that larger $K$ (better performance) means smaller $|W|$ and/or larger $|Q_C|$. Thus the rate of $|Q_C|$ is equal to $K$ times the rate of $|W|$, and is equal to 2.90 times 850 W = 2465 W. This is Joules per second. Per minute it would be 147,900 J.

(b) How many Joules of heat are dumped to the outside hot air per minute? We need first of all the rate of $|Q_H| = $ the rate of $|Q_C|$ plus the rate of $|W| = 2465 W + 850 W = 3315 W$. So in one minute the air conditioner will dump 198,900 J of heat to the outside hot air.

(c) The answers to parts (a) and (b) are not the same, as a matter of fact, the answer to part (b) is necessarily larger than the answer to part (a), because according to the Clausius statement of the second law of thermodynamics, no device can move heat from a colder region to a hotter region without requiring some work (a form of energy transfer) done to it which also gets converted into heat and dumped into the hotter region.

(20.16) Ice making machine. Operates in a Carnot cycle. Takes heat from water at 0.0°C and rejects heat to a room at 24.0 °C. 85.0 kg of water at 0.0°C are converted into ice at 0.0°C.

(a) For a Carnot cycle, $|Q_H| / |Q_C| = T_H / T_C$. Here this ratio is $(273 + 24) / 273 = 1.088$. In this problem, the Carnot cycle is run in the direction to work as a refrigerator. So $Q_C$ is positive (heat extracted from water at 0.0°C and input into the cycle) and $Q_H$ is negative (heat output by the cycle into the air at 24.0 °C in the room). Thus here $Q_C = 85.0 \text{ kg} \times 334 \times 10^3 J/kg = 2.84 \times 10^7 J$. Hence $|Q_H| = 2.84 \times 10^7 J \times 1.088 = 3.09 \times 10^7 J$.

(b) How much energy (work) must be supplied to the device? This the work $W$ done to the device. It is negative since work output is defined as positive. It has the value $|W| = |Q_H| - |Q_C| = 3.09 \times 10^7 J - 2.84 \times 10^7 J = 2.50 \times 10^6 J$.

(20.22) Carnot heat engine. Hot reservoir is a large amount of boiling water at 100 °C. Cold reservoir is a large tub of ice and water (presumably in equilibrium so the temperature is 0 °C.). In 5 minutes of operation, heat rejected by the engine melts 0.0400 kg of ice. How much work is performed? In this problem $Q_H$ is positive (heat input into the engine from the hot reservoir), $W$ is positive (work output by the engine), and $Q_C$ is negative (heat output by the engine into the cold reservoir). Here, $|Q_C| = 0.0400 \text{ kg} \times 334 \times 10^3 J/kg = 1.336 \times 10^4 J$ (in 5 minutes of operation). For Carnot cycle, $|Q_H| / |Q_C| = T_H / T_C$. So $|Q_H| = 1.336 \times 10^4 J \times (373 K / 273 K) = 1.825 \times 10^4 J$. Hence $|W| = |Q_H| - |Q_C| = 1.825 \times 10^4 J - 1.336 \times 10^4 J = 4.89 \times 10^3 J$. This is the work performed by the engine in 5 minutes of operation.

[Note that $|Q_H|$ and $|Q_C|$ must be accurate to four significant digits before $|W|$ can be obtained to three significant digits due to the cancellation of the first digits of $|Q_H|$ and $|Q_C|$. This is a point to notice when you are subtracting two numbers that are very close to each other. If they are closer you might have to keep even more digits in the two numbers that are very close to each other.]
(20.26) Fill tub with 270 kg of 30.0 °C water and warm it by pouring in 5.00 kg of boiling water.

(a) Irreversible, since the initial state is off equilibrium and the process of reaching equilibrium is always irreversible, as heat can only flow from higher temperature to lower temperature.

(b) Let the final temperature be denoted $T_F$. Then it must lie between 100.0 °C and 30.0 °C, as the 30.0 °C water must gain heat as the 100.0 °C water gives out heat. Since no heat has escaped, we must have

$$270 \text{ kg} \times c_{\text{water}} \times (T_F - 30 \text{ °C}) = 5.00 \text{ kg} \times c_{\text{water}} \times (100 \text{ °C} - T_F), \text{ or}$$

$$(270 \text{ kg} + 5.00 \text{ kg}) \times T_F = 270 \text{ kg} \times 30 \text{ °C} + 5.00 \text{ kg} \times 100 \text{ °C}, \text{ giving}$$

$$T_F = 31.27 \text{ °C}. \quad [= 304.42 \text{ K}]$$

[Note that (i) we do not need to know $c_{\text{water}}$ in this calculation, and (ii) there is no need to convert centigrade temperatures to Kelvin (absolute) temperatures here, because only differences in temperatures entered in the set-up equation.]

(c) Calculate the net change in entropy of the system. (Assuming no heat exchange with air or tub.)

We use $dS = dQ/T = mc_{\text{water}} dT/T$. Thus

$$\Delta S_{\text{net}} = \Delta S_{\text{bath water}} + \Delta S_{\text{boiling water}}$$

$$= 270 \text{ kg} \times c_{\text{water}} \int_{303.15 \text{ K}}^{304.42 \text{ K}} \frac{dT}{T} + 5.00 \text{ kg} \times c_{\text{water}} \int_{373.15 \text{ K}}^{304.42 \text{ K}} \frac{dT}{T}$$

$$= 4190 \text{ J/kg·K} \times \left(270 \text{ kg} \times \ln \frac{304.42 \text{ K}}{303.15 \text{ K}} + 5.00 \text{ kg} \times \ln \frac{304.42 \text{ K}}{373.15 \text{ K}}\right)$$

$$= 4729.5 \text{ J/K} - 4264.8 \text{ J/K} = + 464.7 \text{ J/K}.\quad \text{[Note that: (i) The entropy change for the bath water is positive, whereas the entropy change for the boiling water is negative, because the former has absorbed heat whereas the latter has lost heat. Any system that has gained heat has also increased its entropy, whereas any system which has lost heat has also decreased its entropy; (ii) The net change of entropy of “the universe” (i.e., all parts involved) is always positive if a non equilibrium system has reached equilibrium in this process, but an equilibrium system will never become a non-equilibrium system by itself, so the net entropy will never lower itself spontaneously; (iii) The unit of entropy is J/K; (iv) Temperature must be in Kelvin in this calculation; (v) Even though we have derived the entropy formula here, the formula is actually given in the formula sheet, and is } \Delta S = mc \ln (T_{\text{final}}/T_{\text{initial}})\text{, but you must apply it to each sub-system involved. This formula is correct as long as } c \text{ is a constant in the temperature range involved (which is not true in some problems). If } c \text{ does depend on temperature then you must do the integral yourself, because then you cannot take it out of the integral. Thus it is better to see how the formula is obtained. (v) Again because we are evaluate the difference of two close quantities we need to evaluate them to higher accuracy so as to get at least three significant digits in the final answer.]}