Chapter 18, example problems:

(18.06) Identical balloons. Break if $V > 0.900 \text{ L}$. (Note: 1 L = $1 \times 10^{-3} \text{ m}^3$. It is also equal to $1 \times 10^3 \text{ cm}^3$.) $p = 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$. (Note: Pa is just N/m$^2$.)

(a) $T = 22.0 \degree \text{C} = 295 \text{ K}$. Air in balloon.

$$n = \frac{pV}{RT} = \frac{1.013 \times 10^5 \text{ Pa} \times 0.900 \times 10^{-3} \text{ m}^3}{8.314 \text{ J/mol·K} \times 295 \text{ K}} = 0.0372 \text{ mol}$$

(Alternatively, you can do $1 \text{ atm} \times 0.900 \text{ L} / 0.0821 \text{ atm·L/mol·K} \times 295 \text{ K} = 0.0372 \text{ mol}$.)

$$m = nM = 0.0372 \text{ mol} \times 28.8 \text{ g/mol} = 1.07 \text{ g}.$$ 

(b) Helium in balloon. $M = 4 \text{ g/mol}$.

$$m = nM = 0.0372 \text{ mol} \times 4.00 \text{ g/mol} = 0.149 \text{ g}.$$ 

(18.28) Ideal gas @ 27.0 $\degree \text{C} = 300 \text{ K}$ and 1 atm. Assume each molecule at the center of a small cube. Cubes are in contact, forming a lattice.

(a) Length of an edge of a cube $a = \left(\frac{V}{N}\right)^{1/3} = \left(\frac{kT}{p}\right)^{1/3}$

$$= (1.381 \times 10^{-23} \text{ J/molecule·K} \times 300 \text{ K} / 1.013 \times 10^5 \text{ N/m}^2)^{1/3}$$

$$= (4.09 \times 10^{-26} \text{ m}^3)^{1/3} = 3.45 \times 10^{-9} \text{ m} \quad \text{(or 3.45 nm, or 34.5 Å)}$$

(b) Diameter of a typical molecule such as N$_2$ or O$_2 = 3 \text{ Å} = 0.3 \text{ nm}$. The spacing in the gas is about a factor of 10 larger.

(c) Typical molecular spacing in solid $\approx 0.3 \text{ nm}$. The spacing in the gas is about a factor of 10 larger.

(18.44) Water vapor. Tri-atomic molecules. Three translational and three rotational degrees of freedom contributing to the specific heat capacity. Vibrational motion does not contribute (assumption). Molecular mass $M = 18.0 \text{ g/mol}$.

(a) The concept needed here is:

Molar heat capacity = \((R/2) \times (\text{number of translational and rotational degrees of freedom} + 2 \times \text{vibrational degrees of freedom})\)

So for H$_2$O it is \((8.314 \text{ J/mol·K}/2) \times 6 = 24.94 \text{ J/mol·K}\)

Converting it to the specific heat capacity in J/kg·K, we first convert \(M\) to the unit of kg/mol, and then do $c = C/M$:

$$c = 24.94 \text{ J/mol·K} / 18.0 \times 10^{-3} \text{ kg/mol} = 1385.7 \text{ J/kg·K}.$$ 

(b) The result obtained in (a) is smaller than 2000 J/kg·K by 614.3 J/kg·K, which is the same as 11.06 J/mol·K. Dividing it by R/2, we get 2.66. So effectively there are 1.33 vibrational degrees of freedom contributing to the heat capacity of water vapor (at the unspecified particular temperature, presumably just above 100 $\degree \text{C}$ at 1 atm). That is, the vibrational contributions are only partially turned on at this temperature.

(18.46) Molecular speed. Find $T$ such that 94.7% of all molecules have speed < (a) 1500 m/s; (b) 1000 m/s; (c) 500 m/s; Use Table 18.2.

$M_{\text{N}_2} = 28.0 \text{ g/mol} = 28.0 \times 10^{-3} \text{ kg/mol}$. From Table 14.7, 94.7% of all molecules have $v/v_{\text{rms}} < 1.60$.

(a) For $v < 1500 \text{ m/s}$ for this percentage of molecules, we must have
\[ v_{\text{rms}} = 1500 \text{ m/s} / 1.60 = 937.5 \text{ m/s}. \] But \[ v_{\text{rms}} = (3k_B T/m)^{1/2} = (3RT/M)^{1/2}. \]

Hence \[ T = M v_{\text{rms}}^2 / 3R \]
\[ = 28.0 \times 10^{-3} \text{ kg/mol} \times (937.5 \text{ m/s})^2 / (3 \times 8.314 \text{ J/mol} \cdot \text{K}) = 986.7 \text{ K}. \]

(b) For \( v < 1000 \text{ m/s} \) for this percentage of molecules, we must have \( v_{\text{rms}} = 1000 \text{ m/s} / 1.60 = 625 \text{ m/s}. \) Then \[ T = 28.0 \times 10^{-3} \text{ kg/mol} \times (625 \text{ m/s})^2 / (3 \times 8.314 \text{ J/mol} \cdot \text{K}) = 438.5 \text{ K}. \]

(c) For \( v < 500 \text{ m/s} \) for this percentage of molecules, we must have \( v_{\text{rms}} = 500 \text{ m/s} / 1.60 = 312.5 \text{ m/s}. \) Then \[ T = 28.0 \times 10^{-3} \text{ kg/mol} \times (312.5 \text{ m/s})^2 / (3 \times 8.314 \text{ J/mol} \cdot \text{K}) = 109.6 \text{ K}. \]

(18.51) Ice, slowly warmed from very low temperature.

(a) Lowest pressure to observe melting, \( p_1 = p_{\text{tp}} \) (for H\(_2\)O) = \( 0.00610 \times 10^5 \text{ N/m}^2 \) (= \( 0.00610 \times 10^5 \text{ Pa} \)) = 0.00602 atm.

For \( p < p_1 \), only solid to gas phase transition exists.

(b) Maximum pressure \( p_2 \) to still be able to observe boiling transition.

\[ p_2 = p_{\text{critical}} \] (for H\(_2\)O) = \( 221.2 \times 10^5 \text{ N/m}^2 \) (= \( 221.2 \times 10^5 \text{ Pa} \)) = 218.4 atm.

For \( p > p_2 \), only solid to liquid phase transition exists. Liquid can change to gas continuously, without showing a phase transition at a particular (pressure-dependent) temperature.

For \( p_1 < p < p_2 \), both solid to liquid and liquid to gas phase transitions exist, in the sequence: solid \( \rightarrow \) liquid \( \rightarrow \) gas.

(18.60) Flask with a stop cork. \( V = 1.50 \text{ L} = 1.50 \times 10^{-3} \text{ m}^3 \). Filled with C\(_2\)H\(_6\) (ethane gas) at 300 K and 1 atm = \( 1.013 \times 10^5 \text{ N/m}^2 \). \( M_{\text{ethane}} = 30.1 \text{ g/mol} = 3.01 \times 10^{-2} \text{ kg/mol} \).

Warmed to 380 K with stop cork open to atmosphere, then closed cooled back to 300 K.

(a) According to the ideal gas law, at constant volume and number of moles, pressure is proportional to absolute temperature. So \[ \text{Final pressure } p_{\text{final}} = p_{\text{initial}} \times (T_{final} / T_{initial}) = 1 \text{ atm} \times (300 \text{ K} / 380 \text{ K}) \]
\[ = 0.789 \text{ atm}. \]

(b) How many grams of ethane in the flask?

\[ m = nM = (pV / RT)M \]
\[ = (1.013 \times 10^5 \text{ N/m}^2 \times 1.50 \times 10^{-3} \text{ m}^3 / 8.314 \text{ J/mol} \cdot \text{K} \times 380 \text{ K}) \times 3.01 \times 10^{-2} \text{ kg/mol} \]
\[ = 0.001448 \text{ kg} \] (or 1.448 g).

(18.64) person at rest inhales 0.50 L of air with each breath @ 1 atm and 20.0 °C, containing 21\% O\(_2\).

(a) \( N \) (of O\(_2\)) = \( (pV / RT) \times N_A \times 21\% \) (of O\(_2\))
\[ = (1 \text{ atm} \times 0.50 \text{ L} / 0.0821 \text{ atm} \cdot \text{L/mol} \cdot \text{K} \times 293 \text{ K}) \times 6.022 \times 10^{23} \]

molecules/mol \times 21\% = \( 2.63 \times 10^{21} \) molecules (of O\(_2\)).

(b) This man is now resting @ 2000 m elevation. \( T \) is still 20.0 °C. Oxygen percentage and volume inhaled the same. Find again \( N \) (of O\(_2\)).

We see that \( N \propto p \). Pressure is dropped by the factor \( \exp(-mgv / k_B T) = \exp(-M_{\text{O}_2} gv / RT) \)
\[ = \exp (-28.8 \times 10^{-3} \text{ kg/mol} \times 9.81 \text{ m/s}^2 \times 2000 \text{ m} / 8.314 \text{ J/mol} \cdot \text{K} \times 293 \text{ K}) \]
Hence the new number of $O_2$ molecules is $2.63 \times 10^{21} \text{ molecules} \times 0.793 = 2.09 \times 10^{21} \text{ molecules}$

(c) The human body still needs that many $O_2$ molecules per second at high elevations as at sea level, so he/she must breath more heavily or more frequently at high elevations.

(18.68) Insect collisions. Cubic cage 1.25 m on each side Contains 2500 angry bees. Each flying randomly at 1.10 m/s. We model bees as spheres 1.50 cm in diameter. On the average,

(a) Mean distance between collisions $\lambda = \frac{vt_{\text{mean}}}{\sqrt{2}} = (1.25 \text{ m})^3 / (\sqrt{2} \times (0.75 \times 10^{-2} \text{ m})^2 \times 2500) = 0.780 \text{ m. (or 78.0 cm.})$

(b) $t_{\text{mean}} = \frac{\lambda}{v} = 0.780 \text{ m} / 1.10 \text{ m/s} = 0.709 \text{ s.}$

(c) Number of collisions per second $= 1 / t_{\text{mean}} = 1.41 \text{ collisions/s.}$

That this problem is assigned is because insect collisions are similar to molecular collisions in a gas.

(18.74) Hydrogen gas. Volume $V = 5.00 \text{ L}$. Molar mass 2.016 g/mol. Pressure $p = 1.01 \times 10^5 \text{ Pa}$. Temperature $T = 300 \text{ K}$.

(a) Total random kinetic energy $= N \times (\frac{3}{2}) k_B T = n \times (\frac{3}{2}) RT = (\frac{3}{2}) \frac{pV}{RT}$

$= (\frac{3}{2}) 1.01 \times 10^5 \text{ N/m}^2 \times 5.00 \times 10^{-3} \text{ m}^3 = 757.5 \text{ J.}$

(b) If the tank containing the gas is in a swift jet with velocity 300.0 m/s, by hw much is the total K.E. increased?

The total K.E. is increased by $(1/2) m v^2 = (1/2) (2.016 \times 10^{-3} \text{ kg/mol} \times n) \times (300.0 \text{ m/s})^2$, with $n = \frac{pV}{RT} = 1.01 \times 10^5 \text{ N/m}^2 \times 5.00 \times 10^{-3} \text{ m}^3 / 8.314 \text{ J/mol·K} \times 300 \text{ K} = 0.202 \text{ mol.}$ Hence the increased K.E. = 18.37 J. So the percentage increase in K.E. is 2.42%.

(c) No. The temperature does not go up, in spite of the fact that the total K.E. has increased, since the increased K.E. is not associated with random motion.

(18.76) Hydrogen on the sun. Surface temperature $T = 5800 \text{ K}$. Consists largely of hydrogen atoms (not molecules).

(a) $v_{\text{rms}} = \sqrt{(3RT/M)} = \sqrt{(3 \times 8.314 \text{ J/mol·K} \times 5800 \text{ K} / 1.008 \times 10^{-3} \text{ kg/mol})}

= 11979.8 \text{ m/s} = 1.198 \times 10^4 \text{ m/s.}$

(b) Escape speed $= \sqrt{(2GM/R)}$

$= \sqrt{(2 \times 6.6742 \times 10^{-11} \text{ N·m}^2/\text{kg}^2 \times 1.99 \times 10^{30} \text{ kg} / 6.96 \times 10^8 \text{ m})} = 6.178 \times 10^5 \text{ m/s}$

(c) This escape speed is larger than the $v_{\text{rms}}=1.198 \times 10^4 \text{ m/s}$. So very little hydrogen gas can escape the surface of the sun. Nevertheless, a tiny amount of the hydrogen gas can still escape the sun because the Maxwell-Boltzmann distribution function says that there is a tiny fraction of the gas atoms with speed much larger than the rms speed of the gas atoms. Some of these gas atoms are facing upward, and can escape the sun’s surface, effectively cooling the sun by a tiny bit. This is called evaporative cooling, and is used today in some laboratories to cool some trapped atoms to nano-Kelvin temperatures. (The gas is first cooled to already quite low
temperatures by some other method, and is then cooled further by evaporative cooling.)