Chapter 16, example problems:

(16.02) Read this problem carefully. In the first sentence, the information that \( A = 1.2 \times 10^{-8} \text{ m} \Rightarrow p_{\text{max}} = 3.0 \times 10^{-2} \text{ Pa} \) is about Example 16.1. It is not information for this problem. The difference is air vs. water. The information given for this problem is:

Water @ 20 °C. \( B = 2.2 \times 10^9 \text{ Pa} \).
\( v_{\text{sound}} = 1480 \text{ m/s} \).
We still have \( f = 1000 \text{ Hz} \), and \( p_{\text{max}} = 3.0 \times 10^{-2} \text{ Pa} \).
\( A = ? \).

\[ A = \frac{p_{\text{max}}}{Bk} = \frac{3.0 \times 10^{-2} \text{ Pa}}{2.2 \times 10^9 \text{ Pa} \times 2\pi \times 1000 \text{ Hz} / 1480 \text{ m/s}} = 3.21 \times 10^{-12} \text{ m} \] (<< 1.2 \times 10^{-8} \text{ m}).

(This problem has a word error at the very end. It should be compared with 1.2 \times 10^{-8} \text{ m}, not 1.2 \times 10^{-8} \text{ Pa}. The unit of amplitude is “m”, not “Pa”.)

(16.08) Temperature is 27 °C. Longitudinal (sound) waves in:

(a) Hydrogen. \( M = 2.02 \text{ g/mol} \). \( v_{\text{sound}} = \sqrt{\frac{\gamma RT}{M}} \). \( \gamma = 7/5 \) for diatomic molecules.

\[ v_{\text{sound}} = \sqrt{1.40 \times 8.314 \text{ J/mol} \cdot \text{K} \times 300 \text{ K} / 2.02 \times 10^{-3} \text{ kg/mol}} = 1315 \text{ m/s}. \]

(b) Helium. \( M = 4.00 \text{ g/mol} \). \( \gamma = 5/3 \) for monatomic molecules.

\[ v_{\text{sound}} = \sqrt{1.667 \times 8.314 \text{ J/mol} \cdot \text{K} \times 300 \text{ K} / 4.00 \times 10^{-3} \text{ kg/mol}} = 1020 \text{ m/s}. \]

(c) Argon. \( M = 39.9 \text{ g/mol} \). \( \gamma = 5/3 \) for monatomic molecules.

\[ v_{\text{sound}} = \sqrt{1.667 \times 8.314 \text{ J/mol} \cdot \text{K} \times 300 \text{ K} / 39.9 \times 10^{-3} \text{ kg/mol}} = 323 \text{ m/s}. \]

(d) Air. \( M_{\text{ave}} = 28.8 \text{ g/mol} \). \( \gamma = 7/5 \) for diatomic molecules.

\[ v_{\text{sound}} = \sqrt{1.40 \times 8.314 \text{ J/mol} \cdot \text{K} \times 300 \text{ K} / 28.8 \times 10^{-3} \text{ kg/mol}} = 348 \text{ m/s}. \]

(16.18) Note: “sound intensity level” is not the same thing as “sound intensity”. Sound intensity is denoted as \( I \), and has the unit \( \text{W/m}^2 \). Sound intensity level is denoted as \( \beta \), and has the unit of B (bel) or dB (decibel).

(a) \( I = 0.500 \mu \text{W/m}^2 \).

\[ \beta = 10 \text{ dB} \log \left( \frac{I}{I_0} \right) = 10 \text{ dB} \times \log \left( 0.5 \times 10^{-6} \text{ W/m}^2 / 10^{-12} \text{ W/m}^2 \right) = 56.99 \text{ dB}. \]

(b) \( p_{\text{max}} = 0.150 \text{ Pa} \). Temperature \( t = 20 \text{ °C} \).

\[ I = \frac{p_{\text{max}}^2}{2 \rho v} = \frac{0.150 \text{ N/m}^2}{2 \times 1.20 \text{ kg/m}^3 \times 344 \text{ m/s}} = 2.73 \times 10^{-5} \text{ W/m}^2. \]

[If you can not show that \( (\text{N/m}^2)^2 / (\text{kg/m}^3 \times \text{m/s}) \) is just \( \text{W/m}^2 \), then you did not learn what you should have learned in Phys 218.]

\[ \beta = 10 \text{ dB} \times \log \left( 2.73 \times 10^{-5} \text{ W/m}^2 / 10^{-12} \text{ W/m}^2 \right) = 74.35 \text{ dB}. \]

(16.28) Human ear. Auditory canal filled with air. One end open, the other end closed by the eardrum. The eardrum can vibrate, but with an amplitude much smaller than that of the sound wave in the canal. So the eardrum is to a good approximation a node, and the auditory canal is to a good approximation a closed pipe. (See http://ccrma.stanford.edu/courses/150-2001/hearing.html. It is called an open-closed pipe there.) It has \( L = 2.40 \text{ cm} \).

(a) \( \lambda_1 = 4L = 9.60 \text{ cm} \). \( f_1 = 344 \text{ m/s} / 9.60 \times 10^{-2} \text{ m} = 3583 \text{ Hz} \). It is audible.
(b) We need to find the largest odd integer \( n \) such that \( n \times 3583 \text{ Hz} < 20000 \text{ Hz}. \) This \( n_{\text{max}} \) is 5. That is, the highest audible harmonics to this person is the fifth harmonics. This frequency is 17917 Hz.

(16.36) Speakers A and B. Driven by same amplifier in phase. \( f = 172 \text{ Hz} \). Listener 8.00 m from A. Closed distance to B to get destructive interference?
\[ \lambda = \frac{344 \text{ m/s}}{172 \text{ Hz}} = 2 \text{ m}. \]
To get destructive interference, the difference in distances to A and B needs to be 0.5 \( \lambda \), or 1.5 \( \lambda \), or 2.5 \( \lambda \), etc., which means 1 m, 3m, 5m, etc.
Thus the shortest distance to B to get destructive interference is 8 m - 7 m = 1 m.

(16.40) (a) \[ F \rightarrow F + \Delta F. \quad v = \sqrt{F/\mu} \rightarrow \sqrt{[(F + \Delta F)/\mu]} = \sqrt{(F/\mu)}(1 + \Delta F/F)^{1/2} \]
\[ \approx v[1 + (1/2)(\Delta F/F) + \cdots]. \]
\[ f_0 = v/\lambda \rightarrow (v/\lambda)[1 + (1/2)(\Delta F/F) + \cdots] = f_0[1 + (1/2)(\Delta F/F) + \cdots]. \]
Hence, the frequency is shifted by \( f_0(\Delta F/2F) \). This is also the difference of the two frequencies, so it is also the beat frequency.
(b) Beat frequency heard is 1.5 beats per second.
Then \( f_0(\Delta F/2F) = 1.5 \text{ Hz}. \) \( f_0 = 440.0 \text{ Hz}. \) So,
\[ (\Delta F/F) = 0.00682, \text{ or } 0.682 \%. \]

(16.44) Train traveling @ 25.0 m/s. Still air.
Frequency emitted by the locomotive whistle is \( f_0 = 400 \text{ Hz}. \)
(a) Wavelength in front of the locomotive =
\[ (344 \text{ m/s} / 400 \text{ Hz})(1 - 25.0 \text{ m/s} / 344 \text{ m/s}) = 0.798 \text{ m}. \]
(b) Wavelength behind the locomotive =
\[ (344 \text{ m/s} / 400 \text{ Hz})(1 + 25.0 \text{ m/s} / 344 \text{ m/s}) = 0.922 \text{ m}. \]
Frequency heard in front of the locomotive =
\[ 344 \text{ m/s} / 0.798 \text{ m} = 431 \text{ Hz}. \text{ [It is just 400 Hz/(1 - 25.0 m/s / 344 m/s).]} \]
Frequency heard in front of the locomotive =
\[ 344 \text{ m/s} / 0.922 \text{ m} = 373 \text{ Hz}. \text{ [It is just 400 Hz/(1 + 25.0 m/s / 344 m/s).]} \]

(16.64) Steel, density \( \rho = 7.8 \times 10^3 \text{ kg/m}^3 \). Breaking stress = 7.0 \times 10^8 \text{ N/m}^2.
Cylindrical guitar string made of steel. Total mass = 4.00 g.
(a) Tension \( F = 900 \text{ N}. \)
Length of longest string made of this piece of steel?
Radius of the thinnest string made of this piece of steel?
Volume of the string = \( 4.00 \times 10^{-3} \text{ kg} / 7.8 \times 10^3 \text{ kg/m}^3 = 0.513 \times 10^{-6} \text{ m}^3 \).
Cross-sectional area of thinnest and longest string made of this piece of steel = Tension \( F / \text{ breaking stress} = 900 \text{ N} / 7.0 \times 10^8 \text{ N/m}^2 = 1.286 \times 10^{-6} \text{ m}^2 \).
\[ \therefore \text{ Length of longest string made of this piece of steel} = \]
\[ \text{ volume / minimum area} = 0.513 \times 10^{-6} \text{ m}^3 / 1.286 \times 10^{-6} \text{ m}^2 \]
\[ = 0.399 \text{ m}. \]
Radius of the thinnest string made of this piece of steel = \[ \sqrt{(1.286 \times 10^{-6} \text{ m}^2 / \pi)} = 6.40 \times 10^{-4} \text{ m}. \]
(b) Highest fundamental frequency = ?
Longest fundamental-mode wavelength possible = \(2L_{\text{max}} = 0.798 \text{ m}\).
Sound velocity = \(\sqrt{\frac{F}{\mu}} = \sqrt{\frac{900 \text{ N}}{(4.00 \times 10^{-4} \text{ kg}) / 0.399 \text{ m}}}
= 299.6 \text{ m/s}.
Highest fundamental frequency = \(\frac{299.6 \text{ m/s}}{0.798 \text{ m}} = 375.4 \text{ Hz}\).

(16.70) Two identical loudspeakers at A and B. 2.00 m apart.
Driven by same amplifier. \(f = 784 \text{ Hz} \). \(v = 344 \text{ m/s}\).
\(\lambda = 0.439 \text{ m}\). Small microphone moved along BC \(\perp AB\) to P
at distance \(x\) from B. Distance to A is \(\sqrt{(AB)^2 + x^2}\).

(a) Destructive interference:
\(\sqrt{(AB)^2 + x^2} - x = (n + 1/2) \lambda \Rightarrow 4 \text{ m}^2 + x^2 = [x + (n + 1/2) \lambda]^2\)
\(\Rightarrow x = \frac{[4 \text{ m}^2 - (n + 1/2)^2 \lambda^2]}{[2 (n + 1/2) \lambda]}\).
- \(n = 0 \Rightarrow x = 9.007 \text{ m}\).
- \(n = 1 \Rightarrow x = 2.710 \text{ m}\).
- \(n = 2 \Rightarrow x = 1.275 \text{ m}\).
- \(n = 3 \Rightarrow x = 0.534 \text{ m}\).
- \(n = 4 \Rightarrow x = 0.0257 \text{ m}\).
\(n = 5\) or higher \(\Rightarrow\) no solution for \(x\). \([:: \lambda > 2 \text{ m}\].

(b) Constructive interference:
\(\sqrt{(AB)^2 + x^2} - x = n \lambda \Rightarrow 4 \text{ m}^2 + x^2 = [x + n \lambda]^2\)
\(\Rightarrow x = \frac{[4 \text{ m}^2 - n^2 \lambda^2]}{[2n \lambda]}\).
- \(n = 0 \Rightarrow\) no solution, or \(x = \infty \text{ m}\).
- \(n = 1 \Rightarrow x = 4.339 \text{ m}\).
- \(n = 2 \Rightarrow x = 1.840 \text{ m}\).
- \(n = 3 \Rightarrow x = 0.861 \text{ m}\).
- \(n = 4 \Rightarrow x = 0.262 \text{ m}\).
\(n = 5\) or higher \(\Rightarrow\) no solution for \(x\). \([:: 5 \times 0.439 \text{ m} > 2 \text{ m}\].

(c) In order to have no position at all for destructive interference, we want \(n = 0\)
in \(x = \frac{[4 \text{ m}^2 - (n + 1/2)^2 \lambda^2]}{[2 (n + 1/2) \lambda]}\) to already give no solution. That is, we want \((1/2)^2 \lambda^2\) to be already larger than 4 m\(^2\). It means that \((1/2) \lambda\) has to be larger than 2 m. Hence \(\lambda\) has to be larger than 4 m. Then the frequency has to be smaller than \(\frac{344 \text{ m/s}}{4 \text{ m}} = 86 \text{ Hz}\).

(16.74) Ultrasound. Heart wall moving toward the source of ultrasound at some speed \(v_{\text{wall}}\). The frequency received by the moving heart is \(f_0 \left(1 + \frac{v_{\text{wall}}}{v_{\text{sound}}}\right)\). The sound reflected out has then the frequency \(f_0 \left(1 + \frac{v_{\text{wall}}}{v_{\text{sound}}}\right) / \left(1 - \frac{v_{\text{wall}}}{v_{\text{sound}}}\right)\) because the reflected sound is generated by a moving source. This frequency-is higher than the source frequency \(f_0 = 2.00 \text{ MHz}\) by 85 Hz (the beat frequency). Therefore, \((1 + \frac{v_{\text{wall}}}{v_{\text{sound}}} / (1 - \frac{v_{\text{wall}}}{v_{\text{sound}}}) - 1 = 85 \text{ Hz} / 2 \times 10^6 \text{ Hz} = 4.25 \times 10^{-5}\). Since this is very small, we can conclude that \(v_{\text{wall}} / v_{\text{sound}}\) is very small. Then we can approximate \((1 + \frac{v_{\text{wall}}}{v_{\text{sound}}} / (1 - \frac{v_{\text{wall}}}{v_{\text{sound}}}) by \(1 + v_{\text{wall}} / v_{\text{sound}} \times (1 + v_{\text{wall}} / v_{\text{sound}} + \ldots) = (1 + 2v_{\text{wall}} / v_{\text{sound}} + \ldots)\), where we have neglected the square or higher powers of \(v_{\text{wall}} / v_{\text{sound}}\), because small times small is even smaller. (Try 0.1 \times 0.1 or 0.01 \times 0.01. Note here we are talking...
about $10^{-5} \times 10^{-5}$.) Then we find that to a very good approximation we can require $2v_{\text{wall}} / v_{\text{sound}} \approx 4.25 \times 10^{-5}$, which gives $v_{\text{wall}} = 2.125 \times 10^{-5} \times 1500 \text{ m/s} = 0.0319 \text{ m/s}$, or $3.19 \text{ cm/s}$ (as the problem has given $v_{\text{sound}}$ to be 1500 m/s).