# Chapter 34
## Geometric Optics (also known as Ray Optics)
### by C.-R. Hu

## 1. Principles of image formation by mirrors

(1a)  When all length scales of objects, gaps, and holes are much larger than the wavelength of light, we can treat light as a bundle of **light rays**.  Each light ray is a straight line in a uniform medium (i.e., when the index of refraction $n$ is a constant.)  The light rays change direction only when reflection and refraction are encountered.

(1b)  Each **extended object** can be thought of as a collection of points. Each such point can be considered as a point source of light, if it either emits light or is illuminated by a light source. This is because most objects reflect light **diffusely**, that is, in all directions.  Thus any point on an object can be taken as a **point source** of a bundle of light rays pointing in all outward directions that are not blocked.

(1c)  When an eye receives a bundle of light rays that **appears** to all come from one point in space after straight-line extrapolation, then the brain interprets that point as the source of this bundle of light rays.  Thus one can be fooled as to the actual location of the real point source that is emitting the light rays that his eyes receive, **if** the light rays were somehow bent before reaching his eyes.  This can happen when mirrors are involved (or when the index of refraction is not a constant in space, as in the case when one or more interfaces are present, or when there is a gradual change of the index of refraction in a continuous way).  This is when one sees an **image** of an object not located where the object is.

If every point in an image **actually** has a bundle of light-rays crossing through it, then this image is called a **real image**, which can expose a photographic film, if only the film is placed to face the incoming light rays. If **extrapolation** must be involved in order to find the points where bundles of light rays **seem to** all have come from, then the image is called a **virtual image**, which cannot expose a photographic film, but it can be seen.

An image can be **upright** or **inverted** (which means that the image is in  the same orientation or up-side-down relative to the object)**.**  The lateral size of the image can be larger or smaller than that of the original object, by the **lateral magnification $m$ defined as**

$$m = \text{(image size } y' \text{ ) /(object size } y \text{ )}.$$

## 2. Image formation by a plane mirror
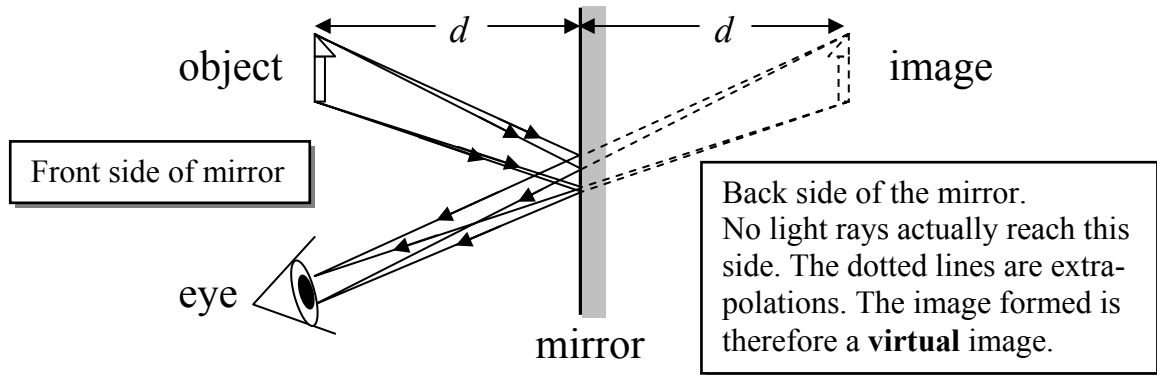


Using the law of reflection, one can conclude that if a bundle of light rays is emitted from a point source which is at a distance $d$ in front of a mirror, then after this bundle of light rays is reflected by the mirror, they will all appear to have come from a point at a distance $d$ behind the mirror.
This point is therefore the **virtual image** of the point source.  Now consider the many points of an extended object in front of a mirror as many point sources of light rays, one can see that the virtual image of this object will be front-back inverted, but not left-right or up-down inverted.  Such an image is said to be **parity-inverted.**  One can convince oneself that the image is of the same size as the real object in this case.  The lateral magnification $m$ is therefore equal to unity for a plane mirror.

Any distance behind the mirror is said to be negative, and any distance in front of the mirror is said to be positive.  Thus if $s$ is the **object distance** from the mirror, and $s'$ is the **image distance** from the mirror, then we have $\boxed{s' = -s}$ for a plane mirror. The minus sign reflects the fact that while the object is in front of the mirror, the image is actually behind the mirror.

When two mirrors are face to face, and an object and an eye are both in the space between the two mirrors, with the eye facing the proper direction, then some light rays from the object can be reflected by both mirrors in succession before reaching the eye. Then the image formed by the first mirror can be treated as the object for the second mirror.  The size and location of this **effective object** is the size and location of the image of the first mirror.  Three or more mirrors can be treated similarly, but they cannot be placed in a straight line, since a mirror will not function if no light rays can reach it.

In all multi-mirror situations, the image formed by the last mirror is the image of the object formed by the combination of mirrors.

## 3. Image formation by a spherical mirror

A spherical mirror is said to be a **convex spherical mirror**, if its reflecting surface is the outer surface of the spherical mirror. That is, if the front of the mirror is facing the outside of the sphere.  Otherwise the spherical mirror is said to be a **concave spherical mirror**.  The radius of a concave spherical mirror is defined to be positive (because its **center of curvature** is in front of the mirror), and the radius of a convex spherical mirror is defined to be negative (because its center of curvature is behind the mirror).

A spherical mirror is usually only made of a small part of a sphere.  A line passing through the **center of the mirror** $V$ and its **center of curvature** $C$ is called the **principal axis** of the mirror.
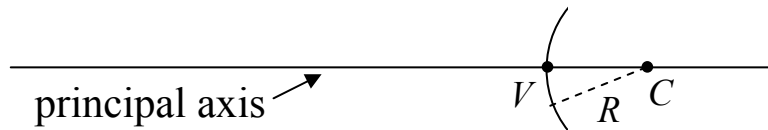
principal axis        $V$   $R$   $C$

If we consider only objects small in comparison with the radius of curvature of the spherical mirror, $R$, lying on or very near the principal axis, and if we work out the image of the object using rays that are nearly parallel to the principal axis, it is called **paraxial approximation**.  Such rays are called **paraxial rays**.  A spherical mirror forms a clear image of an object only under such an approximation.  Thus a spherical mirror should have a size small in comparison with its radius of curvature $R$.  Otherwise the image of a point will not be a point, but will be a short line segment.

Again, we introduce the object distance $s$ (measured from $V$), the image distance $s'$ (also measured from $V$), the object size $y$, and the image size $y'$.  We also introduce a new quantity for a spherical mirror, its **focal length** $f$, which is the distance between $V$ and its focal point $F$.  We define:
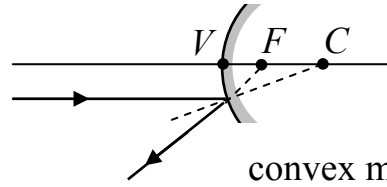
$$f > 0 \text{ for a concave mirror,} \quad \text{and} \quad f < 0 \text{ for a convex mirror.}$$

Then using the law of reflection it can be shown in the paraxial approximation that

$$f = R/2$$        (**focal length of a spherical mirror**).
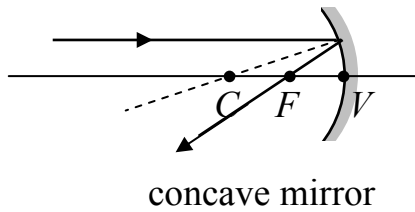
concave mirror                          convex mirror

(**If the mirror appears nearly flat**, *i.e.,* if the size of the mirror is much smaller than its radius, then the focal point $F$ will be exactly half way between the center of the mirror $V$ and the center of curvature of the mirror $C$.  It doesn't appear so in the above figures because the mirrors drawn are not nearly flat.  This is why we need **paraxial approximation**.)

Only two formulas govern the image formation by a spherical mirror:

$$\frac{1}{s'} + \frac{1}{s} = \frac{1}{f} \;,$$

$$m = \frac{y'}{y} = -\frac{s'}{s}\;.$$

Recall that $f = R/2$, and $f$ is positive (negative) for a concave (convex) spherical mirror.  As in the case of a plane mirror, $s$ and $s'$ are positive if in front of the mirror, and negative if behind the mirror.  Also, $y$ and $y'$ are positive if above the principal axis, and negative if below the principal axis.  These equations will let you find the position and size of the image.  To see why they are true (in the paraxial approximation only), and to determine whether the image is real or virtual, upright or inverted, we need:

## 4. Graphical method for mirrors (ray diagram)

The procedure is as follows:

(i) Draw a horizontal line representing **the principal axis**.  Draw a vertical straight line symmetrically crossing the principal axis to represent the spherical mirror. (In the **paraxial approximation**, the mirror surface can be regarded as flat in this graphical method.)  Mark it with a letter $M$ to remind yourself that it is the spherical mirror.  Lightly shade a thin region on one side of the vertical line representing the mirror, to remind you that this is the back side of the mirror.

(ii) Mark the locations of the point $V$ (for the center of the mirror), the point $C$ (for the center of curvature of the spherical mirror), and the point $F$ (for the focal point of the mirror) on the principal axis.  $C$ and $F$ should be in the front (or backside) of the mirror, if the spherical mirror is

concave (or convex). The point $F$ should lie half way between the points $V$ and $C$.

(iii) Draw a short vertical arrow starting on a point on the principal axis pointing upward to represent the object. It should be located where the object is. That is, the whole graph should be drawn to scale. The ratio between the object distance $s$ and the focal length $f$ or the radius of curvature $R$ should be what is given in the problem. Whether the object is in front or in the back (for a **virtual object**) of the mirror should be also as given in the problem. (A real object can only be in front of the mirror, But in a combination of mirrors, the object for the second mirror is the image of the first mirror, which can lie in the back of the second mirror. That is, the light rays can be reflected by the second mirror before they form the image of the first mirror. Such an "object" is called a **virtual object**. In such a case, the object distance $s_2$ for the second mirror would be negative because the "object" is behind the second mirror.

(iv) Starting at the tip of the arrow representing the object, draw at least two of the following three **principal rays**: (a) **The first principal ray** should be parallel to the principal axis. After it is reflected by the mirror, it should go through the focal point $F$, possibly by extension if $F$ is behind the mirror. (b) **The second principal ray** should go through the focal point $F$, possibly by extension if $F$ is behind the mirror. After this ray is reflected by the mirror, it should become parallel to the principal axis. (c) **The third principal ray** should point toward the point $V$. After it is reflected by the mirror this ray should be on the other side of the princepal axis, and making the same angle with the principal axis as that between the incident third ray and the principal axis.

If the graph is drawn to scale, all of these three rays, after reflection, should all go through one point in your graph (possibly after extrapolation), which can then be identified as the image of the tip of the object arrow. Draw now a vertical arrow beginning on a point on the principal axis, and ending on this point that you have just found. This arrow, which might be inverted, is then the image of the object arrow. The size ratio of these two arrows gives you the lateral magnification of the object by this spherical mirror. You can now see whether the image is inverted or upright, by checking whether the image arrow that you just obtained is inverted or upright. You can also check whether the image is real or virtual, by checking whether extrapolation is needed or not, in order to find

the location of the tip of the image arrow, *i.e.*, whether the three principal light rays, after reflection, will need backward extrapolation to the back side of the mirror, before they will intersect at one point.

**Warning:** If you draw your graph poorly, the three rays, after reflection, will not go through a single point, with or without extrapolation. But you will find that they will more or less go through a single point (possibly after extrapolation). Then you should know that your drawing is inaccurate, and you should improve it. In an accurate drawing, these three rays will necessarily go through one point, either in front of the mirror, or in the back of the mirror after backward extrapolation of the three reflected rays. Thus only two rays will be sufficient to find the intersection point.
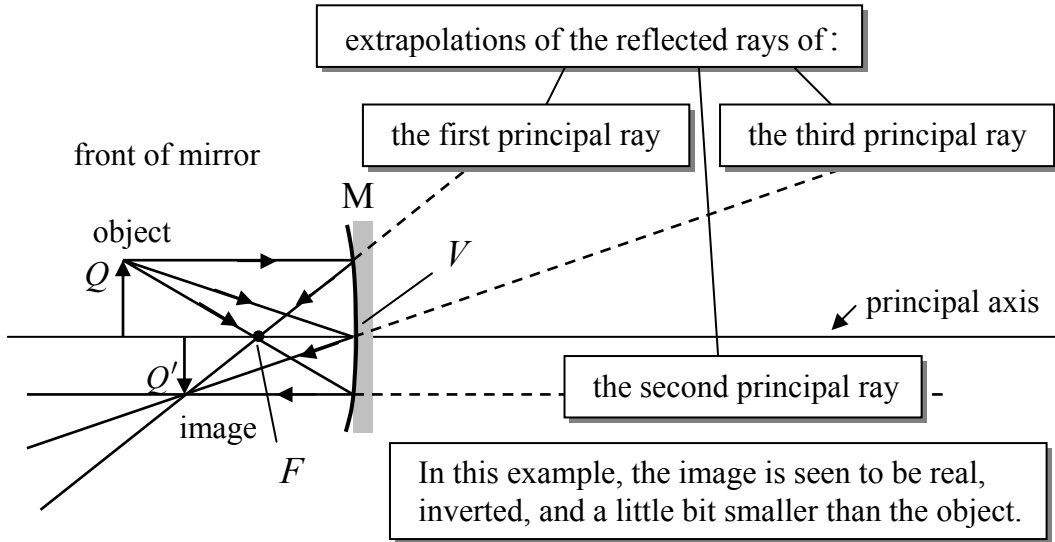
Actually, there is also a **fourth principal ray**: This ray starts at the tip of the object arrow, and goes through the point $C$ (possibly after extrapolation). Its reflected ray will also go through this point $C$, so it will simply trace the incoming ray backward. This is because this ray is perpendicular to the spherical mirror. In an accurate drawing, all four principal rays, after reflection, will go through one point (possibly after extrapolation). But your drawing will unlikely be that accurate, so drawing too many rays might confuse you. All rays that come out of the tip of the object arrow, after reflection by the mirror, will go through the tip of the image arrow (possibly after extrapolation, if the image is virtual), **only if** the size of the mirror is small in comparison to its radius of curvature (so that the mirror can be represented by a straight vertical line in the graph). If the size of the mirror is not that small, then these rays will not all meet at the same point after reflection.

Usually, you need only work out the images of a few extreme points in the object before you can see the location, size, and orientation of the whole image.
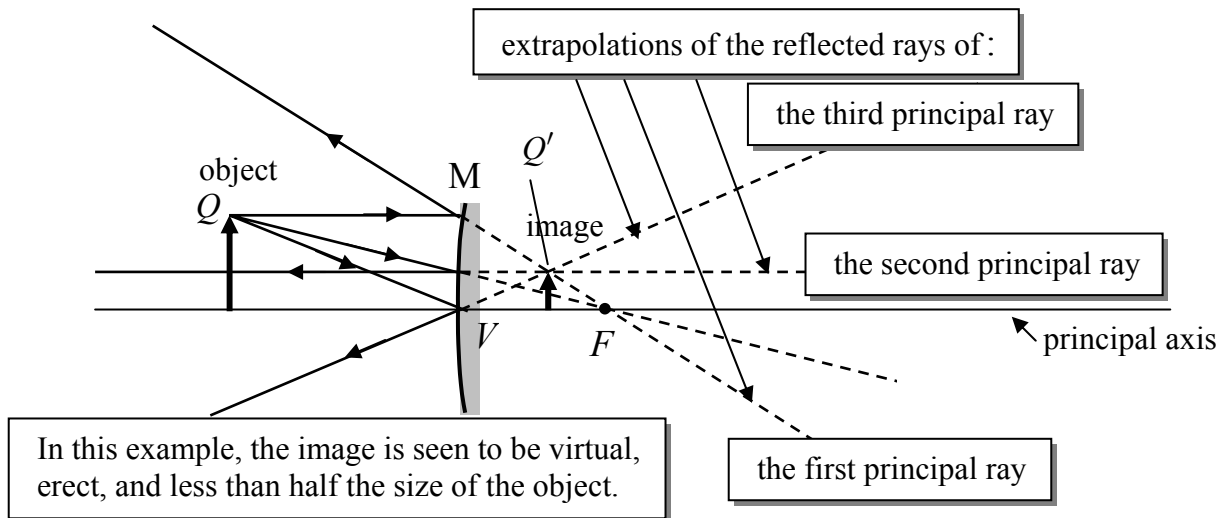
When applying this graphical method, you can draw the mirror as flat and infinitely large, and not worry about any light ray to not hit the actual area of the mirror. This is the spirit of the paraxial approximation. Your purpose is to find the location, size, and characteristics (i.e., upright or inverted, real or virtual) of the image formed only. Actual finite size of the mirror can make you see only a part of the image, depending on the location of the eye seeing it. But this is not why we want to find in this graphical method.

## Examples:

(i) **Concave mirror**  (focal point in front of the mirror, $f > 0$):

extrapolations of the reflected rays of :

the first principal ray

the third principal ray

front of mirror

M

object

$Q$

$V$

principal axis

$Q'$

the second principal ray

image

$F$

In this example, the image is seen to be real, inverted, and a little bit smaller than the object.

(ii) **Convex mirror**  (focal point behind the mirror, $f < 0$):

extrapolations of the reflected rays of :

the third principal ray

object

$Q$

M

$Q'$

image

the second principal ray

$V$

$F$

principal axis

In this example, the image is seen to be virtual, erect, and less than half the size of the object.

the first principal ray

In either example, the fourth principle ray was not drawn.  It starts from the top of the object, and goes toward $O$, which is twice as far from $A$ as $F$.  It is reflected exactly backward.  It also goes through the top of the image (possibly after extrapolation).

Note that only when we can neglect the curvature of the mirror in locating the reflection points can we find all three principal light rays to converge to a single point in the case of a concave mirror, and to diverge as if they all come from a single point in the case of a convex mirror.

# 5. Image formation by a flat interface between two optical media
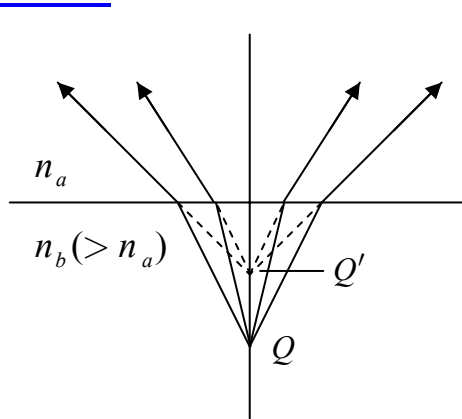


For $n_b > n_a$, the object at $Q$ in the denser medium appears to be closer to the interface (at $Q'$) if viewed on the other side.
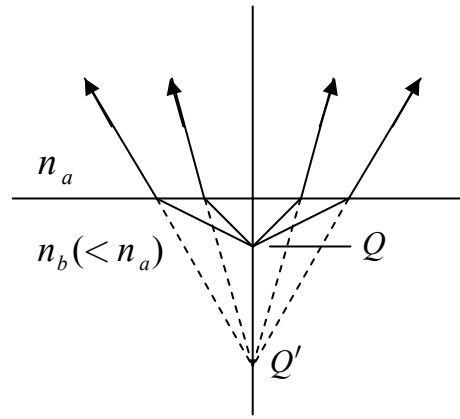
For $n_b < n_a$, the object at $Q$ in the lighter medium appears to be farther from the interface (at $Q'$) if viewed on the other side.

The location of the image can be found by using geometry, trignometry, and the law of refraction. If $s$ is the object distance from the interface, and $s'$ is the image distance from the interface, then it can be easily seen that $s \tan \theta_b = -s' \tan \theta_a$, whereas $n_b \sin \theta_b = n_a \sin \theta_a$. (The reason that $-s'$ appears is because $s'$ itself is negative due to the fact it is obtained by extrapolation. That is, it is not on the side of light. If you are looking in the direction nearly perpendicular to the interface, then both $\theta_a$ and $\theta_b$ will be very small, in which case $\tan \theta_a$ and $\tan \theta_b$ can be approximated by $\sin \theta_a$ and $\sin \theta_b$. Then the two equations can be combined to obtain the simple relation $n_a / s + n_b / s' = 0$. This equation is a special case of a general equation given in the next section.

# 6. Image formation by a spherical interface between two optical media

Such an interface can form a sharp image only if the size of the interface is much smaller than its radius $R$, and all light rays involved are nearly parallel to the principal axis. (This is called **the paraxial approximation**.)

Within this approximation we have:

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}.$$

The situation is shown in the following figure:
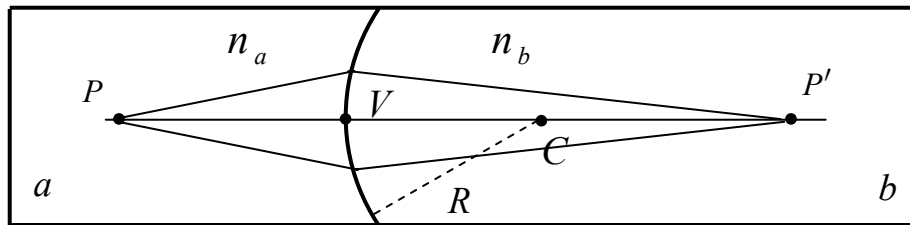


**Note:** (1) $P$ indicates where the object is located. $P'$ indicates where the image is located. We have assumed that light moves from the side $a$ to the side $b$, then we have $n_b - n_a$ on the right hand side of the equation. (2) The object distance $s$ is the distance $PV$. It is positive if $P$ is in side $a$, where light is located <u>before</u> refraction. The next figure shows when $s$ is negative. The image distance $s'$ is the distance $VP'$. It is positive if it is on the side $b$, where light is located <u>after</u> refraction. Then the image (at $P'$) is called a **real image**, and it can expose a photographic film placed there. If extrapolation of the light rays to the $a$ side is needed in order to obtain the image, then $s'$ is negative, and the image is called an **virtual image**. It can not expose a film, since light is actually on the $b$ side. (3) $R$ is positive if the center of curvature of the spherical interface is on the side $b$. It is negative if it is on side $a$. That is, looking along the direction of light, if the interface appears convex (concave), then $R$ is positive (negative).

**When can $s$ be negative?** Foe any **real object**, $s$ is always positive. However, the image of a previous optical stage can become the object of the present stage. If light from a previous stage attempts to form an image at $P$ as in the following figure, but is intercepted by the spherical interface, so the image is actually formed at $P'$, then the image of the previous stage is never formed at $P$, and it is called a **virtual object** for the present stage. The object distance $s$ is then negative, with a magnitude equal to $VP$. Note that extrapolation is again involved.
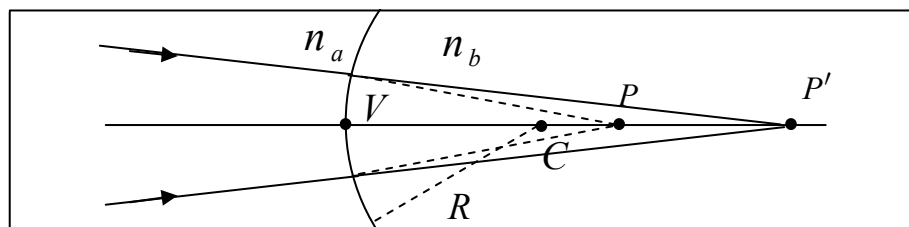
The size of the image $y'$ (positive for an upright image and negative for an inverted image), is related to the size of the object $y$ (positive for an upright object and negative for an inverted object) by the **magnification factor** $m$:

$$m \equiv \frac{y'}{y} = -\frac{n_a s'}{n_b s}.$$

Proof will be omitted, but it is not difficult to see why $n_a s'$ is in the numerator, and $n_b s$ is in the denominator. (Large $n_b$ or $s$ at fixed $n_a$ and $s'$ tends to make the image smaller). There is a minus sign in the front, since in the simplest situation, when both $s$ and $s'$ are positive, the image is actually inverted, so $y'$ is actually negative if $y$ is positive. (A negative $m$ means that the image is inverted relative to the object. A positive $m$ means that the image is not inverted relative to the object.)

Putting $R = \infty$, and you get the situation of a flat interface. But notice that the formulas in this section are only valid in the paraxial approximation. So all light rays must be nearly parallel to the principal axis. That is, they must all be **paraxial rays**.

## 7. What is a lens?

Beginning with a circular platelet of a uniform transparent material with an index of reflection $n > 1$, grind and polish its two flat surfaces into either convex or concave spherical surfaces. If used in vacuum or in a medium with an index of reflection $n' < n$, and if the final product is thicker in the middle, it is a **converging lens**. If it is thinner in the middle, it is a **divergent lens**. (The opposite is true if $n' > n$.)

A convergent lens can still have the following different shapes:
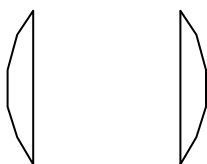


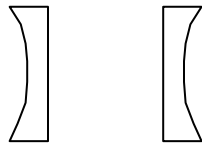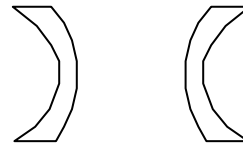double convex      planoconvex      convex meniscus

and a divergent lens can still have the following different shapes:

double concave          planoconcave          concave meniscus

A convergent lens can bend parallel light rays toward a single **focal point** $F$ on the **exit side** of light (left figure).  A divergent lens can bend parallel rays into a divergent beam, <u>as if</u> all light rays on the **exit side** of light came from a single focal point $F$ on the **entrance side** of light (right figure).  Since no light actually came from this point, it is called a **<u>virtual focal point</u>** of this divergent lens.
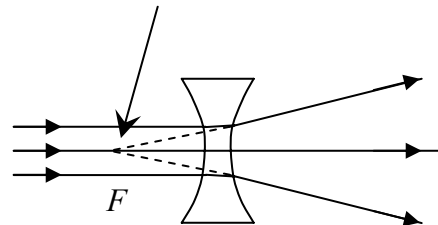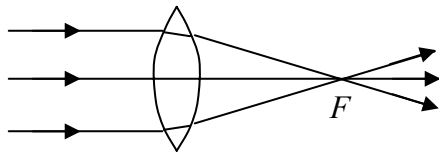


# 8. <u>Thin lens approximation</u>

<u>If the thickness of a lens and its lateral size are both much smaller than the radii of its two faces</u>, one can treat it as if it has no thickness, and is also (essentially) flat.  Then all incident light rays can be treated as if they are bent (refracted) at a planar surface of no thickness representing that lens.  The center of that planar surface is the **center of the lens** $O$.  The distance between $O$ and the **focal point** $F$ is the **focal length** $f$ of this lens.  <u>The focal length $f$ is positive if the lens is convergent, when $F$ is on the **exit side** of light.  The focal length is negative if the lens is divergent, when (the vertual focal point) $F$ is on the **entrance side** of light.</u>
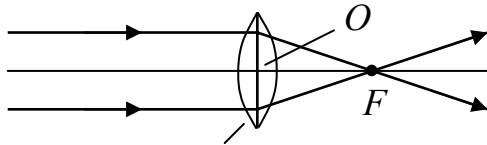


convergent lens
($f = \overline{OF} > 0$)
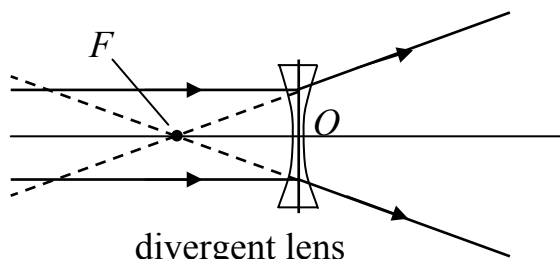
divergent lens
($f = -\overline{OF} < 0$)

Since light can come in from either side of a thin lens, <u>a thin lens (whether convergent or divergent) has two focal points, one on each side, and at the same focal length $f$ from the center $O$ of the thin lens</u>.  Only one of the two

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focal points is used in each actual image formation, since light must be incident from one side of the lens.  You should always use the focal point on the exit side of light if the lens is convergent, and you should always use the focal point on the entrance side of light if the lens is divergent.

## 9. Image formation by thin lenses

The two equations

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \; ,$$

and

$$m \equiv \frac{y'}{y} = -\frac{s'}{s} \; ,$$

are again valid for obtaining the location, type, and size of the image.  However, the sign convention is somewhat different from the case of spherical mirrors:

The object distance $s$ is positive if it is on the **entrance side of light**.  (All real objects will therefore automatically have a positive $s$.  But the image of a first lens can become the object of a second lens. In that case, the "object" of the second lens may not be on the entrance side of light to the second lens. In that case the $s_2$ for the second lens can then be negative.)

The image distance $s'$ is positive if the image is on **the exit side of light**, and it is negative if the image is on the entrance side of light.  That is, if the image is formed by extrapolating the emerging light rays on the exit side backward into the entrance side of the lens, then $s'$ is negative.  In this case the image is not on the same side as the emerging light.  It is then a **virtual image**.  No film can be exposed by a virtual image. Only a **real image** can expose a film.

## 10. Lensmaker's equation

A lens, whether convergent or divergent, is made of a uniform optic material with an index of reflection $n > 1$, and with two spherical surfaces of radii $R_1$ and $R_2$. ($R_1$ is radius of the first surface encountered by the incident light.)  These radii are positive if they appear convex to the incoming light.

The **lensmaker's equation** is:

$$\frac{1}{f} = (n-n')(\frac{1}{R_1} - \frac{1}{R_2}) \, , \qquad \text{(for thin lenses only)}$$

where $n'$ is the index of refraction of the surrounding medium. (If the lens is used in air, you can approximately put $n' = 1$.)  The equation can be derived by applying the interface equation to both surfaces of the lens. It does not apply to thick lenses, because then distances are measured from different $O$ for the two surfaces.  One might as well use the interface equation twice in that case, one for each surface of the lens.

If this lens is used in a medium with $n' > n$, then $(n - n') < 0$, and a convergent lens in air becomes a divergent lens in this medium, and a divergent lens in air becomes a convergent lens in this medium, because the sign of $f$ gets changed by the negative sign of $(n - n')$!

The **power of a lens** is $P = 1/f$.  The unit of power in the SI unit system is diopter (D).  That is, $1 \text{ D} = 1 \text{ m}^{-1}$.  If $f = 2$ m, then P = 1 / (2 m) = 0.5 D.
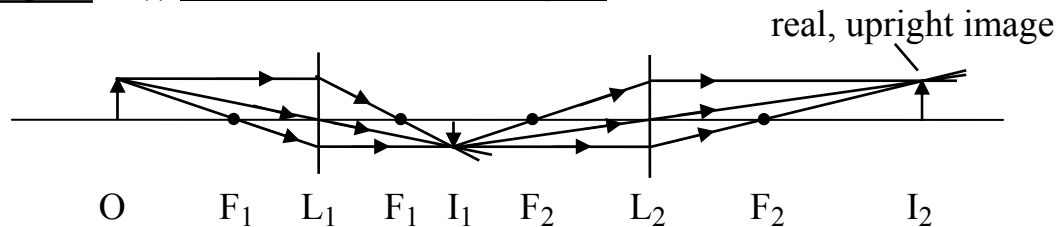
## 11. <u>Graphical method for thin lenses (ray diagram)</u>

Much like the graphical method for spherical mirrors, except that the three principal light rays are now defined as follows:  (i) **<u>The first principal light ray</u>** is parallel to the optic axis before it is refracted by the lens.  After going through a convergent lens, it will change its direction to point toward the focal point on the exit side of light. If it has gone through a divergent lens, then it will point in a direction to look like it is coming from the (virtual) focal point on the entrance side of light.  (ii) **<u>The second principal light ray</u>** (or its extension) goes through the focal point that is on the entrance (exit) side of light if the lens is convergent (divergent).  After it has gone through the lens, it will change its direction to become parallel to the optic axis.  (iii) **<u>The third principal light ray</u>** points toward the center of the lens.  After going through the lens, it will not change direction for either a convergent lens or a divergent lens.  Note that the centers of curvature of the two surfaces of the lens should not be used in the graphical method for thin lenses, and therefore <u>there is no "fourth principal light ray" for thin lenses</u>, unlike the case of spherical mirrors.  Again, <u>the image of the first lens can be the "object" of the second lens.</u>  If this "**object**" for the second lens is not on the entrance side of light for the second lens, then the object length $s_2$ for the second length is negative for such a case. It is then called a **virtual object.**
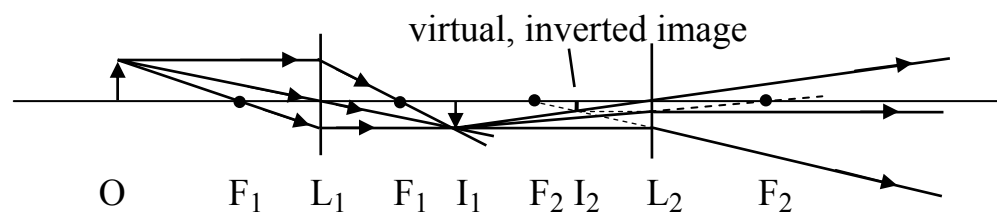
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Helpful suggestions:  Use the notations $s_1$, $s_1{}'$, and $f_1$ for the first lens, and use $s_2$, $s_2{}'$, and $f_2$ for the second lens.

You should replace all thin lenses by straight lines (actually flat planes) perpendicular to the optic axis, and let all changes of directions of light rays occur on these lines (plans).

**Examples:**   (i) **Both lenses are convergent:**

real, upright image

$$\text{O} \qquad \text{F}_1 \quad \text{L}_1 \quad \text{F}_1 \ \text{I}_1 \quad \text{F}_2 \quad \text{L}_2 \quad \text{F}_2 \qquad \text{I}_2$$

(ii) **First lens is convergent, second lens is divergent:**

virtual, inverted image

$$\text{O} \qquad \text{F}_1 \quad \text{L}_1 \quad \text{F}_1 \ \text{I}_1 \quad \text{F}_2 \ \text{I}_2 \quad \text{L}_2 \qquad \text{F}_2$$

# 12. Cameras

Uses only one convergent lens ($f > 0$), which can be moved to change $s'$ (the distance between the lens and the film), and $s$ (the distance between the lens and the object), without changing $s + s'$ (the distance between the object and the film).

**Shutter speed** --- to control the amount of light reaching the film. Too much light leads to **overexposure**. Too little light leads to **underexposure**.

$$\boxed{f\text{-number} = f\,/\,D} \qquad (f = \text{focal length}, \ D = \text{aperture diameter.})$$

If $f$- number = 2, it is given on a camera as $f/2$.  Standard $f$– number values are 2.0, 2.8 (=$2\sqrt{2}$), 4.0, 5.6 (=$4\sqrt{2}$), 8.0, 11 (=$8\sqrt{2}$), 16, 22 (=$16\sqrt{2}$), 32.  These numbers are called $f$ **-stops**.

When the $f$– number goes <u>down</u> by a factor of $\sqrt{2}$, D goes <u>up</u> by a factor of $\sqrt{2}$.  The area of the lens then goes up by a factor of 2.  The amount of light entering the lens then doubles.  The smallest $f$–number of a lens is referred to as the speed of the lens.  (Why?  Because the smallest $f$–number uses the

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full size of the lens, and the smaller $f$–number is, the faster you can take a picture.)  Larger $f$–numbers are obtained by setting the aperture smaller (by controlling the **iris diaphragm**).

**Zoom lens** ━ uses a collection of lenses with variable spacing to allow continuous control of the effective $f$ of the combination.

**Telephoto lens** ━ has a larger $f$ than the normal lens, so the image can be larger.  (Because $m = -s'_i/s$, and $s' \sim f$ when $s$ is large.)

**Wide-angle lens** ━ has a shorter $f$ than the normal lens, so the image can be smaller ━ to fit a bigger view in the film.  (Because the size of the film is fixed, to fit a bigger picture in it, you want the image to be smaller. So $f$ should be smaller.)

**Projector**  ━ a camera run in reverse, with the locations of object and image inter-changed. This can be done to any lens combination, and is called **the principle of time-reversal invariance.** This is because the Maxwell equations obeys time reversal invariance (with the direction of $\vec{B}$ inverted at the same time).  That is, the path of all light rays can be reversed, as if future and past have been inter-changed.

## 13. Human eyes; corrective lenses

**Pupil** ━ The size of a circular area at the center of an eye lens that is not blocked (by the **iris**) for light to enter the eye ball.

**Retina** ━ A layer on the back side of an eye ball where optic nerves are located.

**Nearsightedness (or myopia)** ━ An eye-ball that is too long so that the focal point of the eye lens is in front of the retina. Such an eye is said to be **myopic**.

**Farsightedness (or hyperopia)** ━ An eye-ball that is too short so that the focal point of the eye lens is behind the retina.  Such an eye is said to be **hyperopic**.

**Correcting glasses** ━ A divergent lens of the appropriate focal length placed in front of the eye can correct a myopic eye.  A convergent lens of the appropriate focal length placed in front of the eye can correct a hyperopic eye.  Treat such problems as a combination of two lenses.

**Astigmatism:** Two mutually perpendicular principal planes can be found, each containing the optical axis of the eye lens, such that light rays in those two planes are focused to different focal points. This single eye lens thus has two focal lengths! This is because the eye lens does not have spherical surfaces, but rather has elliptical surfaces. To correct such an eye defect, a lens with compensating ellipticity must be used.

Eye lens
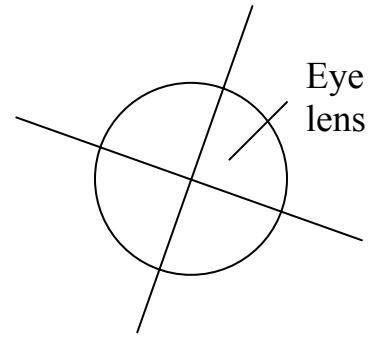
# 14. Magnifying glass (Magnifier)

A magnifying glass is just a convergent lens. However, what one now cares about is the **angular magnification** $M$. Its definition is as follows:

To use a magnifying glass, one usually puts the object at its focal point, so that $s = f$. Then $s'$ will be negative infinity, and the image will be **virtual** and **erect**, and infinitely far away from the eye (so that the eye can be in the relaxed state).
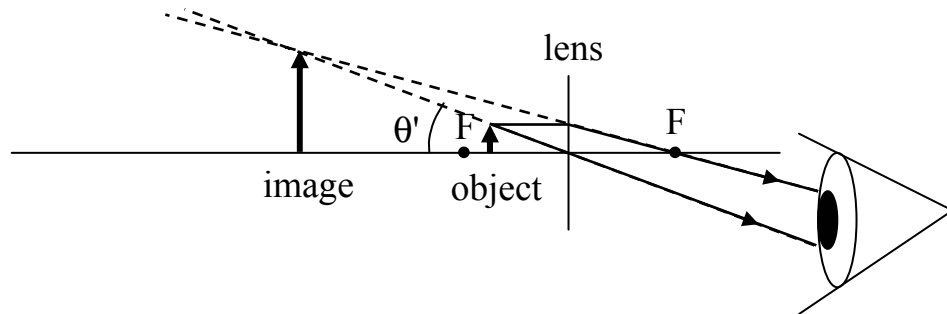


**The angular size of the image** is $\theta' = y'/|s'| = y/s$ . Here we have $s' = -\infty$ , and $s = f$. Thus $\theta' = y/f$. On the other hand, if the magnifying glass is not used, the object would have to be placed at the closed distance that the eye can see clearly (with maximum strain), which is $\boxed{N = 25 \text{ cm}}$ for a **normal eye**. Thus the angular size of the object would be $\theta = y/N$, if the magnifying glass is not used. Comparing the two angular sizes, we obtain **the angular magnification of the magnifying glass**:

$$\boxed{M = \frac{\theta'}{\theta} = \frac{y/f}{y/N} = \frac{N}{f}} \quad \left[ \begin{array}{l} \text{For relaxed eye.} \\ \text{which is focused} \\ \text{at infinity.} \end{array} \right]$$

or

$$M = \frac{\theta'}{\theta} = \frac{y/s}{y/N} = \frac{N}{s} = N\left(\frac{1}{f} + \frac{1}{N}\right) = \frac{N}{f} + 1.$$

(For maximally-strained eye, which is focused at the near point.)

## 15. Telescopes

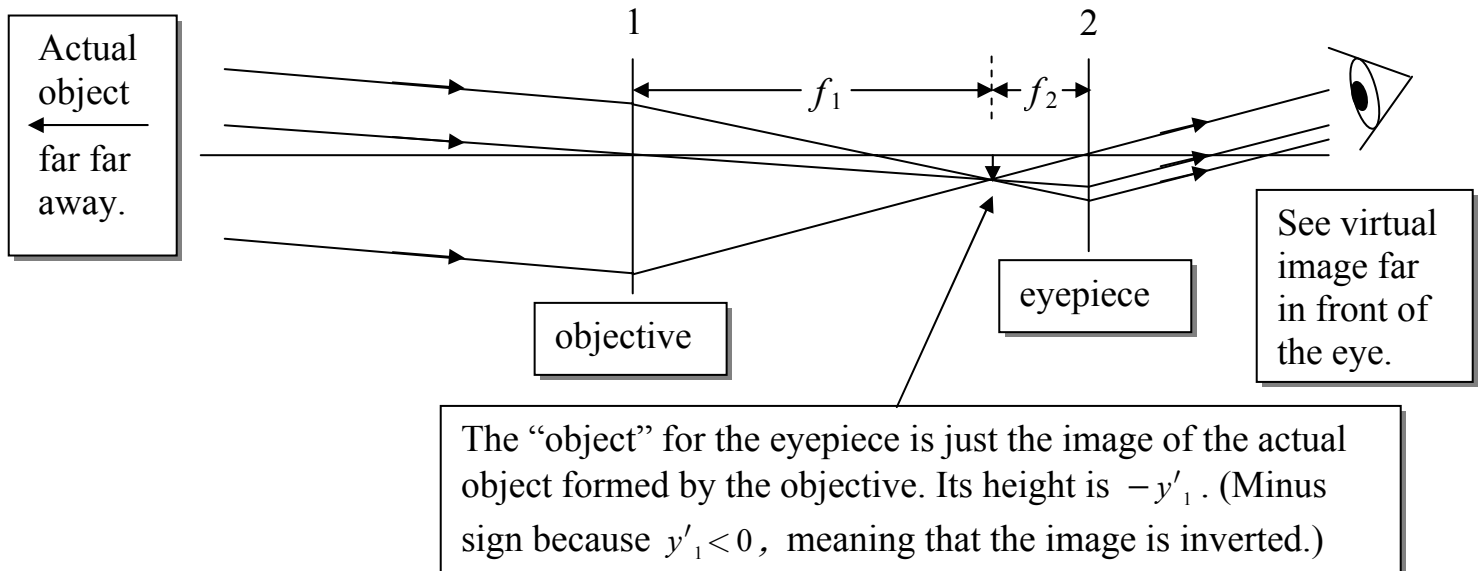**Astronomical** (or **refracting**) **telescope** — uses two convergent lenses: An **objective** lens with $f_1 > 0$, and an **eyepiece** lens with $f_2 > 0$. For seeing objects at infinity with a relaxed eye, the final image formed by the two lenses must also be at infinity. — The image formed by the objective must be at distance $f_1$ from the objective, and at distance $f_2$ from the eyepiece, and lies between them.

**Total (angular) magnification power** of the telescope:

$$M = \frac{\theta'}{\theta} = \frac{-y_2/f_2}{y_1'/f_1} = -\frac{f_1}{f_2} \qquad \text{(because } y_2 = y_1'\text{).}$$

(Minus sign indicates that the "object" for the eyepiece, which is just the real image of the actual object formed by the objective, is inverted, so the final virtual image of the object formed by the eyepiece is also inverted.)



Actual object far far away.

1

$f_1$ $f_2$

2

objective

eyepiece

See virtual image far in front of the eye.

The "object" for the eyepiece is just the image of the actual object formed by the objective. Its height is $-y'_1$. (Minus sign because $y'_1 < 0$, meaning that the image is inverted.)

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**Astronomical <u>reflecting</u> telescope** — uses a concave mirror as the objective, and uses either "a flat mirror plus a convergent lens" or a diverging mirror as the eyepiece.

**Terrestrial telescope** — to view objects on earth, must provide upright images. — **The Galilean design** uses a divergent lens as the eyepiece (2), at distance $f_1 - f_2$ from the objective (1), so that the image formed by the objective is at the exit side of light for the eyepiece, and is therefore a **virtual object** to the eyepiece, which means that $s_2 < 0$. A second design (called a **spyglass**) uses three convergent lenses. Adding prisms to fold the light rays in order to reduce the total length of the telescope, one has the **prism binocular**. It is a pair of telescopes for the two eyes.

## 16. <u>Compound microscope</u>

For seeing very small objects at close distance. Also uses two convergent lenses: An **objective** lens with $f_1 > 0$, and an **eyepiece** lens with $f_2 > 0$. An <u>inverted virtual image</u> is formed by the lens-combination at infinity for a relaxed eye. The **overall magnification** of the microscope is the product of the **objective magnefication** $m_1 = y_1' / y_1 = s_1' / s_1 = (l - f_2) / s_1 \approx l / f_1$, and the **eyepiece (angular) magnification** $M_2 = N / f_2$, i.e.,

$$M = m_1 M_2 = \left( \frac{N}{f_2} \right) \left( \frac{l - f_2}{s_1} \right) \approx \frac{N l}{f_2 f_1},$$

where $l$ is the distance between the objective and the eyepiece.

## 17. <u>Aberation of lenses and mirrors</u>

**Spherical aberration** — When a lens or mirror is not small in comparison with the radius or radii involved, it will not be able to focus all parallel light rays to a single focal point. Changing spherical mirrors to parabolic mirrors solve this problem. Correction for this problem in lenses also exist, but is more complicated, and involves multiple of lenses of different indices of reflection.

**Off-axis aberrations** — Additional aberrations for off-axis objects for large lens or mirror.

**Chromatic aberration** — Caused by the frequency dependence of $n$, so light of different colors will have different focal lengths. May be corrected

by using a suitable doublet of lenses of different *n*, one convergent and one divergent. Note that spherical mirrors have no chromatic aberration, which is why objectives of large telescopes use spherical mirrors rather than lenses**.**

## 18. Corrective glasses for near- or far-sightedness

If a **relaxed eye** is **near-sighted**, then its far point is not at infinity, but is at some finite distance *D* from the eye. The person then needs to wear a **divergent corrective lens** which, for an object at infinity, can form a <u>virtual</u> image of it at the far point of the eye in order for the eye to see it clearly.  If a <u>relaxed</u> eye is **far-sighted**, then an object at infinity will form an image by this <u>relaxed</u> eye on a point behind its retina. Only a <u>virtual object</u> behind the eye lens can form a real image of it by this eye on its retina. Thus this relaxed eye needs to wear a **convergent corrective lens**, which, for an object at infinity, will form an image behind the eye lens. It would then become the virtual object of the eye lens at where it needs to be, so the relaxed eye can see it clearly.  <u>So far, the correction is on a relaxed eye.</u>  A person older than about 45 to 50 years of age can have an eye lens which has lost some of its elasticity.  Thus even if the eye's far point is at infinity which requires no correction, its near point *N* can still be larger than 25 cm. This eye then needs to wear a <u>convergent</u> <u>reading glass</u>, which, for an object at 25 cm, will form a <u>virtual</u> image of it at *N*, so the eye can see it clearly.

## 19. Relation with the next two chapters

**Geometric optics**, which treats light as a collection of light rays, will no longer give right answers if the sizes of objects, gaps, and holes, etc., are comparable to or smaller than the light wavelength.  Light will no longer go in straight lines, and very interesting and fundamental **interference** phenomena will occur.  Light passing through two parallel slits will produce a set of many parallel bright lines on a screen; A beautiful color pattern can appear on a very thin oil film illuminated by white light; A thin transparent coating, with a thickness equal to a quarter of the light wavelength, can eliminate reflection. These and other related phenomena can have very useful applications, such as measuring the speed of light to a high precision, or confirming Einstein's theory of relativity, or determining the precise atomic arrangement in a solid.  A related concept is called **diffraction.**  It can explain why an optical microscope can never let you resolve separations less than about 50 nm.  It can also limit the performance of telescopes and cameras. Geometric optics gives us a <u>false hope</u> that magnification has no limit.  **Physical (or wave) optics** tells us why it can't be true.