Chapter 35 Examples

35.4

\[ \begin{array}{ccc}
A & B & C \\
X & X & f, \lambda \text{ can be varied} \\
\hline
120 \text{ m} & 40 \text{ m}
\end{array} \]

**a)** longest \( \lambda \) for destructive interference at \( A \).

\[ \frac{120 \text{ m} + 40 \text{ m}}{\lambda} - \frac{40 \text{ m}}{\lambda} = \frac{1}{2} \text{ or } 120 \text{ m} = \frac{\lambda}{2} \]

Hence \( \lambda_{\text{longest}} = 240 \text{ m} \)

**b)** longest \( \lambda \) for constructive interference at \( A \).

\[ \frac{120 \text{ m} + 40 \text{ m}}{\lambda} - \frac{40 \text{ m}}{\lambda} = 1 \text{ or } \lambda = 120 \text{ m} \]

35.16 \[ \lambda = 624 \text{ nm} \] \[ \theta_1 = 11.0^\circ \text{ for dark fringe} \]

**a)** \( d \sin \theta_1 = \frac{1}{2} \lambda \)

\[ d = \frac{\frac{1}{2} \times 624 \text{ nm}}{\sin 11.0^\circ} = 1635 \text{ nm} \]

(Or 1635 \( \mu \)m.)

**b)** total number of bright fringes \( = 2n_{\text{max}} + 1 \),

where \( n_{\text{max}} \) corresponds to the largest \( \theta_{\text{max}} < 90^\circ \) satisfying \( d \sin \theta_{\text{max}} = n_{\text{max}} \lambda \). To find it, note that \( d \sin 90^\circ = 1635 \text{ nm} = 2.63 \lambda \). Thus we must have \( n_{\text{max}} = 2 \) and \( 2n_{\text{max}} + 1 = 5 \).

The total number of dark fringes is either \( 2n_{\text{max}} \) or \( 2n_{\text{max}} + 2 \) depending on whether there are two dark fringes outside the outermost two bright fringes. Since 2.62 > 2.50, this is indeed the case. So the answer is \( 2 \times 2 + 2 = 6 \).
Two antennas separate \( d = 9.00 \text{ m} \) radiate in phase \( f = 120 \text{ MHz} \).

\[
\begin{align*}
\downarrow & \quad x = 150 \text{ m} \\
\uparrow & \quad x = 150 \text{ m} \\
\end{align*}
\]

Intensity \( I_0 \) of one receiver so it becomes 1.8 m closer to one antenna than to the other.

\( \phi \) (phase difference) = \( 2\pi \times \frac{1.8 \text{ m}}{2} \)

\[
= 2\pi \times \frac{1.8 \text{ m}}{3 \times 10^8 \text{ m/s} / 1.20 \times 10^8 \text{ s} / \text{m}} = 4.524 \text{ radians}
\]

(or \( 259.2^\circ \))

\( \phi = \frac{\pi}{2} \text{ rad} \), \( \sin \phi = 2\pi \frac{4.5 \sin \theta}{\lambda} \approx 2\pi \frac{d \theta}{\lambda} = 2\pi \frac{dy/R}{\lambda} \),

we find \( \frac{d}{2} = \frac{2\pi}{\lambda} \frac{dy}{R} \) or \( y = \frac{2R}{\lambda d} \)

or \( y = \frac{3 \times 10^8 \text{ m}}{1.20 \times 10^8 \text{ s} / \text{m}} \times \sqrt{150 \text{ m}^2 - (4.5 \text{ m})^2} \\ 4 \times 9.00 \text{ m} = 10.4 \text{ m} \).

Actually, \( \sqrt{150 \text{ m}^2 - (4.5 \text{ m})^2} \) can be approximated by 150 m here.

The figure to help you solve this problem is:

\[
\begin{align*}
\downarrow & \quad d \\
\uparrow & \quad \theta \\
\end{align*}
\]

\( d \sin \theta \sim d \theta \) (in rad.)

\( \theta \sim \frac{y}{R} \).
glass plate 9.00 cm long

\[ \lambda = 656 \text{ nm} \] illuminated from above.

How many interference fringes per cm?

We first find the spacing between the bright fringes, call it \( \Delta x \). Then

\[ \Delta x \cdot \theta = \Delta y \quad \text{but} \quad 2 \Delta y = \lambda, \quad \text{and} \quad \theta = \frac{0.008}{9}. \]

So \( \Delta x = \frac{656 \text{ nm}}{2} \cdot \frac{9}{0.008} = 0.000369 \text{ m} \)

( or 0.369 mm, or 369 \( \mu \text{m} \) )

Michelson interferometer \( \lambda = 606 \text{ nm} \)

Jan moves the movable mirror away by \( \Delta y \).

Counts 818 fringes moving →

Linda

Now uses \( \lambda' = 502 \text{ nm} \) and moves the movable mirror toward the observer \( \lambda' \), counts 818 fringes moving ←.

a) Since light is reflected by the movable mirror, any \( \Delta y \) is traveled twice, so \( 2 \Delta y = 818 \Delta \)

or \( \Delta y = 409 \cdot \lambda = 2.479 \times 10^{-5} \text{ m} = 2.479 \times 10^{-4} \text{ m} \)

Also \( \Delta y' = 409 \lambda' = 2.053 \times 10^{-5} \text{ m} = 2.053 \times 10^{-4} \text{ m} \)

b) Resultant displacement of movable mirror

\[ = 2.979 \times 10^{-4} \text{ m} - 2.053 \times 10^{-4} \text{ m} \]

\[ = 0.926 \times 10^{-4} \text{ m} \text{ or } 9.26 \text{ mm} \]
Two speakers, \( d = 42.2 \text{ cm} \), \( f = 1570 \text{ Hz} \)

The cable length difference \( \Delta \) signal reaches one (A) 0.15\( \text{ m} \) before the other (B). \( v = 330 \text{ m/s} \).

Sound detected far away, \( T = \frac{1}{f} = 0.00637 \text{ s} \).

\[
\Delta = \frac{0.159 \text{ m}}{0.637 \text{ ms}} \times 2\pi = 0.250 \times 2\pi
\]

In the figure above, sound from speaker B will delay further by \( (d \sin \theta / 2) \times 2\pi \).

To get maximum intensity, we need constructive interference. That is, we need the total delay to be integer multiples of \( 2\pi \). So we require

\[
\frac{4.5 \sin \theta}{\lambda} + 0.250 = 1, 2, 3, \ldots
\]

but \( d / \lambda = 42.2 \text{ cm} / (330 \text{ m/s} / 1570 \text{ Hz}) \)

\[
= 2.008 \quad \text{and} \quad \sin \theta \leq 1
\]

so the left-hand side cannot be larger than 2.258. Thus for the right-hand side, we can only take 1 and 2, giving

\[
\theta = 21.9^\circ \quad \text{and} \quad 60.6^\circ
\]

Next, we consider negative \( \theta \). The left-hand side can go as low as \( -2.008 + 0.250 = -1.758 \).

So for the right-hand side, we can only take 0 and -1, giving

\[
\theta = -7.15^\circ \quad \text{and} \quad -38.5^\circ
\]

These are all angles for maximum intensity.
35.48 Two slit interference, different widths.
Amplitude of first wave: $E_1$,
Amplitude of second wave: $2E_1$,
Phase difference: $\phi$. Intensity $I(\phi)$?

a) 

By law of cosine

$$E_{\text{final}} = E_1^2 + 4E_1^2 + 4E_1^2 \cos \phi$$

(check: for $\phi = 0$, it gives $E_{\text{final}} = 3E_1 \checkmark$)

$$E_{\text{final}} = 5E_1^2 + 4E_1^2 \cos \phi$$

$: I_{\text{final}}(\phi) = \text{constant} \times \left( \frac{5}{9} + \frac{4}{9} \cos \phi \right)$

maximum at $\phi = 0$, when the right hand side becomes just the constant. So by definition it can be denoted as $I_0$.

We then obtain $I_{\text{final}}(\phi) = I_0 \left( \frac{5}{9} + \frac{4}{9} \cos \phi \right)$.

b) Minimum at $\phi = \pi$, whence $\cos \phi = -1$,

and $I_{\text{minimum}} = I_0 / 9$.

Graph:

$$I_{\text{final}}$$

$I_0$

$I_0 / 9$

$0 \quad \pi \quad 2\pi$ $\phi$
\[ \lambda_1 = 700 \, \text{nm}, \quad \lambda_2 \text{ unknown} \]

3rd bright fringe from \( \lambda_1 \) matches a dark fringe from \( \lambda_2 \). So

\[ d \sin \theta = 3 \lambda_1 = (n + \frac{1}{2}) \lambda_2 \]

[Explanation: Same \( \theta \) on the screen correspond to the same \( \theta \) of the light rays coming out of the two slits.]

Thus

\[ \lambda_2 = \frac{3}{n + \frac{1}{2}} \lambda_1. \]

\( n = 0 \) gives \( \lambda_2 = 4200 \, \text{nm} \)
- not visible, not acceptable.

\( n = 1 \) gives \( \lambda_2 = 1400 \, \text{nm} \)
- not visible, not acceptable.

\( n = 2 \) gives \( \lambda_2 = 840 \, \text{nm} \)
- not visible, not acceptable.

\( n = 3 \) gives \( \lambda_2 = 600 \, \text{nm} \)
- visible, acceptable.

\( n = 4 \) gives \( \lambda_2 = 467 \, \text{nm} \)
- visible, acceptable.

\( n = 5 \) gives \( \lambda_2 = 382 \, \text{nm} \)
- not visible, not acceptable.

No need to try larger \( n \), since it will give even smaller \( \lambda_2 \).

\( \therefore \lambda_2 \) can only be 467 nm and 600 nm.

We don't need to know \( d \).
White light, normal incidence on glass plate \((n = 1.52)\), air above and below. Constructive interference in reflection for \(\lambda_1 = 477.0 \text{ nm} \) and \(\lambda_2 = 540.6 \text{ nm} \) as the next longer wavelength. Thickness \(t = ?\)

- \(n = 1\): Reflection from top surface
- \(n = 1.52\): Has a \(\pi\) phase shift since it is by a denser medium, not at the bottom surface.

Hence to get constructive interference, we need \(2t = (n + \frac{1}{2}) \lambda_1\). Thus

\[(n + \frac{1}{2}) \lambda_1 = 2t = (n - 1 + \frac{1}{2}) \lambda_2\]

\[n = \frac{\frac{1}{2} (\lambda_1 + \lambda_2)}{\lambda_2 - \lambda_1} = \frac{1}{2} \frac{1017.6}{63.6} = 8\]

Hence \(t = \frac{1}{2} \times 8.5 \times 477.0 \text{ nm} = 2027.25 \text{ nm}\)

\(\approx 2.03 \text{ \(\mu\)m}\)

Explanation: Next higher \(\lambda\) for constructive interference corresponds to \(n\) lowered by 1.
35.58  Newton's rings

D for third bright ring 0.850 mm.

Fill space with water \( n = 1.33 \)

New diameter \( D' = ? \)

Interference is due to reflection from the bottom of the lens (no phase shift) and from the top of the glass plate (or phase shift). So for the 3rd bright ring we must require

\[
2t = (2.5)\lambda
\]

\[
2R (1 - \cos \theta) = 2.5 \lambda
\]

with \( R \sin \theta = D/2 \), \( \cos \theta = \sqrt{1 - (D/2R)^2} \)

Thus

\[
2R (1 - \sqrt{1 - (D/2R)^2}) = 2.5 \lambda
\]

\[
2R - \sqrt{4R^2 - D^2} = 2.5 \lambda
\]

\[
4R^2 - D^2 = (2R - 2.5 \lambda)^2 = 4R^2 - 10.0 R \lambda + 6.25 \lambda^2
\]

or

\[ 10.0 R = D^2 + 6.25 \lambda^2 \approx D^2 \quad (\therefore D \gg \lambda) \]

\[
R = \frac{D^2}{10.0 \lambda} \quad \text{which is 0.850 mm}
\]

Now fill the gap with water. \( \lambda \Rightarrow \lambda/n \)

\[
R' = nR' = 1.33 \times 0.850 \text{ mm} = 1.13 \text{ mm}
\]

\[ \rightarrow \text{New diameter of the third ring} \]