PHYSICS-218
EXAMINATION 3

- Name: _______________________
- Student ID: ___________________
- Section number: ___________________
(I). The small mass \( m \) sliding without friction along the looped track shown in Fig. 1 is to remain on the track at all times, even at the very top of the loop of radius \( r \).

(a) Calculate, in terms of the given quantities, the minimum release height \( h \). Next, if the actual release height is \( 3h \), calculate \( (6\text{pts}) \)

(b) the normal force exerted by the track at the bottom of the loop, \( (6\text{pts}) \)

(c) the normal force exerted by the track at the top of the loop, \( (6\text{pts}) \)

(d) the normal force exerted by the track after the block exits the loop on the the flat section. \( (6\text{pts}) \)

Fig. 1  Problem I

(a). First, look at the dynamics at the top of the loop. Assume that the normal force is \( N \), we have

\[
mg + N = ma_c = m \frac{v^2}{r}
\]

This implies that the minimum \( v \) at the top is given by \( v = \sqrt{rg} \), which occurs when \( N = 0 \).

Now, we can apply the conversation of energy: we have \( E_i = mgh \) and \( E_f = \frac{1}{2}mv^2 + mg(2r) \), and from \( E_i = E_f \), we have \( h = \frac{5}{2}r \).

(b). Now we have \( E_i = mg(3h) = 7.5mg \text{r} \), and \( E_f = \frac{1}{2}mv^2 \), and hence \( E_i = E_f \)
implies that \( v^2 = 15r \text{g} \).

At the bottom, \( N - mg = ma_c = m \frac{v^2}{r} = 15mg \), and hence \( N = 16mg \).

(c). Now, \( E_f = \frac{1}{2}mv^2 + 2mg \text{r} \), so the conversation of energy implies that \( v^2 = 11r \text{g} \).
Use the first equation, we have \( N = 10mg \)

(d). \( N = mg \)
(II) A uniform thin machine part is a flat circular plate of radius \( R \) that has a circular hole of radius \( \frac{1}{2}R \) cut out of it. The center of the hole is a distance \( \frac{1}{2}R \) from the center of the plate, Fig. 2. What is the position of the center of mass of the plate? (15pts)

\[
\begin{align*}
\frac{m}{M} &= \frac{\pi \left( \frac{R}{2} \right)^2}{\pi R^2 - \pi \left( \frac{R}{2} \right)^2} \\
&= \frac{1}{3}
\end{align*}
\]

Let \( x \) denotes the center of mass for the disk with the hole, then

\[
0 = \frac{m \left( \frac{1}{2}R \right) + M x}{m + M}
\]

Thus \( x = -\frac{R}{3} \).

(III) The Hoover Dam was constructed on the Colorado River, to regulate water and generate electricity. In such a hydro-electric power plant, what type of energy is converted to produce electricity? (5pts)

Gravitational potential energy. It is alright with answers such as kinetic energy or mechanical energy.
(IV). A uniform disk of mass $M$ and radius $R$ starts rolling (from rest) down a $30^\circ$ incline of length $L$. (Assume that $R << L$.)

(a) If the disk rolls without slipping, what will be its speed at the base of the incline? (8pts)

(b) What will be its total kinetic energy at the base of the incline? (8pts)

(c) What minimum value must the coefficient of static friction have if the disk is not to slip? (10pts)

![Fig. 4 Problem IV](image)

(a) There is more than one way to solve this problem. One is to draw force diagram and find out the acceleration. Let $N, f$ denote the normal force and the friction respectively, then we have

$$M g \sin \theta - f = M a, \quad f R = I \alpha, \quad N = M g \cos \theta.$$  

In the second equation, the torque is calculated with respect to the axis passing the center of mass, in which case, the gravity force and normal force contributes no torque. The disk is rolling without slipping, it follows that $a = R \alpha$. Thus we have

$$a = \frac{M g \sin \theta}{M + I/R^2} = \frac{2}{3} g \sin \theta.$$  

It follows that $V = \sqrt{2 a L} = \sqrt{\frac{4}{3} g L \sin(\theta)} = \sqrt{\frac{2}{3} g L}$.

(b). $K = \frac{1}{2} M V^2 + \frac{1}{2} I \omega^2 = \frac{1}{2} (M + \frac{I}{R^2}) V^2 = \frac{1}{2} M g L$  

This result should be expected since it is precisely the same as the initial total gravitational potential energy. In fact one can perfectly use the conservation of energy to answer the above two questions.

(c) From the first 4 equations, one can find out $f = \frac{1}{3} M g \sin \theta$. Thus we have

$$\mu_s^{\text{min}} = \frac{f}{N} = \frac{1}{3} \tan \theta = \frac{1}{3 \sqrt{3}}.$$  

In solving this problem, you are expected to know $\sin(30^\circ) = \frac{1}{2}$ and $\tan(30^\circ) = \frac{1}{\sqrt{3}}$, and $I = \frac{1}{2} M R^2$. 

(V). A block of mass \( m = \frac{1}{4}M \) slides (from rest) down a 30° incline of height \( h \). At the bottom, it strikes a block of mass \( M \), which is at rest on the horizontal surface, Fig. 4. (Assume a smooth transition at the bottom of the incline.) If the collision is elastic, and friction can be ignored, determine

(a) the speeds of the two blocks just after the collision, and (10pts)
(b) how far back up the incline the smaller mass will go. (10pts)
(c) What is the upper limit on the mass \( m \) if it is to rebound from \( M \), slide up the incline, stop and slide down the incline, and collide with \( M \) again? (10pts)

![Fig. 5 Problem V](image)

(a). Before collision: we have \( V_m = \sqrt{2gh} \) and \( V_M = 0 \). It follows that after the elastic collision, their respective speeds become

\[
V'_m = \frac{m - M}{m + M} V_m = -\frac{3}{5} V_m = -\frac{3}{5} \sqrt{2gh}, \quad V'_M = \frac{2m}{m + M} V_m = \frac{2}{5} V_m = \frac{2}{5} \sqrt{2gh}
\]

Here we have substituted \( m = \frac{1}{4}M \).

(b). Having obtained \( V'_m \), we can easily obtain the new height by using the conservation of energy

\[
\frac{1}{2} m V'_m^2 = m g h'
\]

Thus we have \( h' = \frac{9}{25} h \).

(c). Clearly, if \( V'_m \) is bigger than \( V'_M \), then the mass \( m \) can catch and collide with the mass \( M \). Since we have

\[
V'_m = \frac{m - M}{m + M} V_m, \quad V'_M = \frac{2m}{m + M} V_m,
\]

it follows that the requirement is given by

\[
\left| \frac{V'_m}{V'_M} \right| = \left| \frac{m - M}{2m} \right| \geq 1.
\]

The equality occurs when \( m = \frac{1}{2}M \), and it is straightforward to verify that the inequality holds for \( m < \frac{1}{2}M \).

Note that it also has a solution of \( m = -M \) for the equality to occur. This solution is not physical since there is no negative mass.