Feynman's Path-Integral Approach for Intense-Laser–Atom Interactions

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Atoms interacting with intense laser fields can emit electrons and photons of very high energies. An intuitive and quantitative explanation of these highly nonlinear processes can be found in terms of a generalization of classical Newtonian particle trajectories, the so-called quantum orbits. Very few quantum orbits are necessary to reproduce the experimental results. These orbits are clearly identified, thus opening the way for an efficient control as well as previously unknown applications of these processes.

Quantum mechanics has been successful in explaining the physics of the microworld. Its standard mathematical formulation, which provides the basis for calculations of quantum-mechanical processes (1), requires a nearly complete break with classical intuition. In contrast, the alternative formulation of quantum mechanics developed by Feynman in terms of path integrals builds on the familiar Lagrangian concept of the action of an orbit in space and time and appears to be much closer to classical concepts (2). In Feynman’s formulation, the probability amplitude of any quantum-mechanical process can be represented as a coherent superposition of contributions of all possible spatio-temporal paths that connect the initial and the final state of the system. The weight of each path is a complex number whose phase is equal to the classical action along the path. Even though this approach turned out to be very useful

Reference and Notes

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in quantum field theory, it has, nevertheless, received much less practical use, due in part to the large number of different paths required to describe most phenomena (3).

Recently, the path-integral interpretation has set the frame for a unified view of the physics of nonlinear laser-atom interactions. Indeed, some phenomena that take place in intense fields (4) have only recently been partly elucidated. For example, in the process of above-threshold ionization (ATI) an atom may absorb many hundred more photons than necessary to get ionized, ejecting an electron of very high energy (more photons than necessary to get ionized, the process of above-threshold ionization) has been partly elucidated. For example, in quantum field theory, it has, neverthe-

sub-femtosecond regime (4) and ultraviolet (XUV) for which water is transparent. Such high-harmonic generation (HHG) can reach and exceed 300 \( \frac{A}{\text{nm}} \), i.e., the region of the light spectrum between x-ray and ultraviolet (XUV) for which water is transparent. Such high-harmonic generation (HHG) is making available brilliant table-top sources of coherent XUV radiation, with pulse durations potentially in the sub-femtosecond regime (9). Other phenomena of interest are the nonsequential multiple ionization (creation of a multiply charged ion within a fraction of the laser-field period) (10, 11) and photodetachment in external fields (12). The path integral formalism suggests we envision these processes in terms of “quantum orbits,” i.e., space-time trajectories of the participating electrons. These quantum orbits have, however, imaginary parts related to tunneling ionization that determine the probability of the process. Quantum orbits have been able to explain subtle features of HHG (13) and ATI (14).

We report experimental and theoretical results that demonstrate the pervasive presence of quantum orbits as the basic building blocks for the probability amplitudes of these processes. Despite the complexity of the highly nonlinear laser-matter interaction, a very small number of clearly identified quantum orbits is sufficient to describe ATI or HHG processes. We observe the same type of orbits at work in ATI and HHG. Thus, in ATI spectra generated by a laser with elliptical polarization, each part of the spectrum can be attributed to one particular pair of quantum orbits. Moreover, we show how the contribution of a single quantum orbit can be isolated in phase-matched HHG, allowing for a direct measurement of the corresponding classical action. For both cases, we present experimental evidence in parallel with the theory. These experiments have pushed the present-day technology to its limits.

A description of ATI and HHG processes is given by the strong-field approxima-

\[ k(t_f - t) = \int_0^\tau d\tau eA(\tau) \]  

\[ [k - eA(t)]^2 = 2mE_0 \]  

This completes the reduction of the path integral to a sum over a few orbits.

The three above equations enforce energy conservation in the process of tunneling, return of the electron to the position from where it started, and energy conservation either when it rescatters into a state with the final (drift) momentum \( p \) or recombinates to produce a harmonic photon with energy \( \hbar \omega_y \), respectively. The solutions of the saddle-point equations are complex, because Eq. 3 cannot be satisfied for any real time \( E_0 < 0 \). Hence, the orbits described by Eq. 2 are complex as well. The solutions can be ordered according to increasing travel time \( \tau_n = \tau_{n-1} - \tau_n \) so that \( \tau_n < \tau_{n-1} \) when \( n < n' \). Equations 1 through 5 illustrate that the amplitude \( M \) is the coherent superposition of the contributions from these complex quantum orbits. In general, only those with travel times shorter than one or two periods of the field will make noticeable contributions to the sum described by Eq. 1.

Qualitatively different interference patterns occur, depending on how many and which trajectories are involved. For an experimental demonstration of the role of various orbits, the conditions should be chosen such that the contributions of different quantum orbits are easily distinguishable; i.e., they should appear in different energy regions and/or be of different strengths. This is most clearly observed in ATI experiments, which measure single-atom effects, and in particular with the use of an elliptically polarized laser field (14, 19). In contrast to linear polarization, where the high-order ATI spectrum consists of one single plateau with its cutoff near 10 \( \frac{U_p}{(5, 6)} \) (\( U_p \) is the ponderomotive energy, proportional to \( It/d \), where \( I \) is the laser intensity), a staircase with several steps is observed for elliptical polarization. The results of such a measurement (Fig. 1) (20) display the rich structure of the spectrum predicted by the quantum-orbit approximation described by Eqs. 1 through 5 (19). The scheme of these theoretical predictions is represented by the straight lines.

The top step in the spectrum does not concern us here: it is made up by the “direct” events, i.e., those that leave the ion without rescattering. For the remaining steps, each can be attributed to the particular pairs of quantum orbits identified in the figure. The physical origin of the steps is as follows: in the process of tunneling, the electron most likely starts its orbit with zero velocity after
tunneling. For elliptical polarization, however, in order that such an orbit return to the parent ion, its starting velocity must be non-zero. The larger it is, the smaller the contribution of the associated orbit. Such is the case for the shortest orbits (depicted in blue), which require a particularly large starting velocity. Theory also predicts a small step (shown in green) in between the two main ones; the experimental data may show an indication of it.

Thus, elliptic polarization leads to a particular richness of the electronic quantum orbits and offers the possibility of microscopic control at the single-atom level of both ATI and HHG (21). Measured HHG, however, is generated by an ensemble of atoms. Therefore, it offers a unique possibility of macroscopic control of the emission and, thereby, of isolating the contribution of a single trajectory, as shown below. The HHG spectrum emitted by a single atom irradiated by a linearly polarized laser field exhibits a characteristic plateau for harmonic energies below \(\sqrt{E_p} + 3.2U_p\) and a rapidly decreasing cutoff region for higher energies. The upper-plateau region is dominated by the pair of orbits with the shortest and next-to-shortest travel times \(\tau_1\) and \(\tau_2\) (15–17), as similar to ATI (orbits drawn in dark and light blue). The contributions of these two trajectories to the atomic polarization have different laser-induced phases, related to the action accumulated along the orbit \(\tau_n(\xi)\) (\(n = 1, 2\)) of Eq. 2. We can approximate the real part of this action, \(ReS_{\text{act}}\), by the product \(-U_p\tau_n\) of the ponderomotive energy by the travel time \(\tau_n\). The interference of the contributions of these two orbits results in an irregular single-atom HHG spectrum. However, the harmonic field measured in the experiments is the coherent addition of the fields produced by the different atoms. Efficient HHG requires phase matching of the harmonic field with the driving polarization.

It has been shown recently that, in the case of a guided propagation, the atomic dispersion can compensate for the “geometric” dispersion (equivalent to the Gouy phase of a focused laser beam), resulting in good phase matching (22). If we focus a laser tightly into the atomic medium, the geometric dispersion is increased and no longer balanced by the atomic dispersion. However, now the laser-induced, position-dependent \(U_p\tau\) phase of the atomic polarization is an important term involved in phase matching. In the longitudinal direction, this symmetric term (relative to the focus position) competes with the antisymmetric geometric phase. If we locate the atomic jet behind the focus, they may compensate each other if \(U_p\tau\) does not vary too rapidly, i.e., for the \(\tau_1\) trajectory. Locating the atomic jet before or at the focus degrades phase matching on axis. However, it may lead to efficient phase matching off the propagation axis, provided the radial phase gradient is large enough and associated with the \(\tau_2\) trajectory. Thus, phase matching should provide a powerful mean to isolate and identify the contributions of the individual quantum orbits. Such macroscopic control of the coherence properties of HHG has been predicted in (23) and observed in spatial measurements.

Here, we concentrate on the spectral properties of the emission and show an unambiguous separation of the contributions from the two trajectories. Indeed, the laser-induced \(U_p\tau\) phase inevitably leads to a phase modulation (chirp) of the harmonic emission because \(U_p\) is time-dependent after the laser envelope, such that the harmonic spectrum exhibits a signature of the \(\tau_1\) and \(\tau_2\) trajectories. At the same time, the experiments reported in (24, 25) provide indirect signatures of quantum trajectories. In a series of recent experiments, however, we have accomplished a direct characterization of these trajectories.

The first experiment is complementary to the one of Bellini et al. (13), in which the spatial resolution of the contributions from the \(\tau_1\) and \(\tau_2\) trajectories was reported. We measured the distortion of the harmonic spectral profiles as a function of the jet-to-focus location, which acts as a phase-matching parameter (Fig. 2). This evolution can be explained within the frame of the above theory: the short \(\tau_1\) trajectory leads to a small, laser-induced chirp, i.e., a narrow harmonic spectrum (shaded in dark blue). Moving the atomic jet from after to before the focus gradually increases the contribution of the long \(\tau_2\) trajectory, which causes a larger chirp and, thus, a broader spectrum, shaded in light blue. Our experimental study ruled out other phenomena that can also induce a spectral distortion such as a blueshift of the laser frequency due to atomic ionization (26), self-phase modulation of the laser beam in the nonlinear medium, or any mechanism that only depends on the laser intensity. However, there is still a possibility that a temporal narrowing of the emission, e.g., due to transient phase matching, could induce such spectral broadening.

To answer this question, we needed to measure jointly the temporal and the spectral profiles of the harmonic emission. This problem, which is particularly challenging for XUV pulses, was solved by coupling an XUV monochromator to an electron spectrometer (27). Both spectral width and pulse duration increase as the jet is moved from the after to before the focus (Fig. 3). Thus, we conclude that the spectral-width increase is indeed due to the laser-induced chirp, which varies with z along with the increasing contribution of the \(\tau_2\) trajectory (large chirp).

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**Fig. 1.** ATI spectrum in xenon for an elliptically polarized laser field with ellipticity \(\xi = 0.36\) and intensity \(7.7 \times 10^{13}\) W/cm\(^2\) for emission at an angle with respect to the polarization axis as indicated. The different steps of the spectrum are shaded in different colors. For each step, the responsible quantum orbits, calculated from Eqs. 2 through 5, are displayed in shades of the same color. The crosses mark the position of the parent ion, its starting velocity must be non-zero. The larger it is, the smaller the contribution of the associated orbit. Such is the case for the shortest orbits (depicted in blue), which require a particularly large starting velocity. Theory also predicts a small step (shown in green) in between the two main ones; the experimental data may show an indication of it.

**Fig. 2.** Spectral profiles of the 19th harmonic generated in a 1-mm-long neon jet by a 130-fs 800-nm laser pulse focused (f/80) at \(4 \times 10^{14}\) W/cm\(^2\). From bottom to top, the jet is moved from after to before the focus, as indicated in the inset. The color shades, light and dark blue, refer to the orbits of Fig. 1.

**Fig. 3.** (A) Spectral width \(\Delta\lambda\), (B) pulse duration \(\Delta t\), and (C) the product \(\Delta t \Delta \nu\) for the 19th harmonic generated in argon as a function of the phase-matching parameter \(z\). The 70-fs laser is focused to \(2 \times 10^{14}\) W/cm\(^2\).
Density Fluctuations Under Confinement: When Is a Fluid Not a Fluid?

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20. The acquisition of a single spectrum requires the recording of up to a billion photocounts. Therefore, femtosecond lasers with extremely high repetition rates should be used, and the acquisition data must be able to handle electron count rates of 200 kHz and more. The laser system used for the ATI experiment delivers 50-fs pulses with a repetition rate of 100 kHz and intensities up to 3 ¥ 1014 W/cm2. The photocounts are recorded with a high-resolution time-of-flight spectrometer.  
21. For the theory of HHG by an elliptically polarized laser field, see (29) for an early experiment, see (30). Similar control is offered by the use of multicolor fields or static magnetic fields; see, for instance (31).  
24. This laser-induced chirp can be compensated by the imposed chirp of the laser. Such control is demonstrated in (32).  
25. Shaping the temporal envelope of the harmonic emission results in a corresponding shaping of the spectral profile; see (33).  
27. The harmonics generated in an argon jet are sent into an XUV monochromator. By moving the grating in the first diffraction order and closing the exit slit, we measure a well-resolved spectrum. Then, the grating is set in the 0th order (so that the XUV pulse is not time-stretched), the slit is widely opened, and all the harmonic components are directed into a xenon gas target in the source volume of an electron spectrometer, together with a second laser beam, where they produce two-color ATI (harmonic + second laser with variable delay). The corresponding photoelectrons give a cross-correlation signal from which the harmonic pulse duration can be derived, as shown in (34).  
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Storage of light within a small volume is essential for the understanding of a vast array of common problems in science, such as biological interactions, fracture propagation, and molecular tribology and adhesion, as well as pressure solvation and other geophysical processes. When a fluid is confined, its phase behavior is altered and excluded-volume effects become apparent. Pioneering measurements performed with the surface forces apparatus have revealed so-called structural or oscillatory solvation forces as well as the occurrence of a finite shear stress, which was interpreted as a solidification transition. Here, we report measurements obtained with an extended surface forces apparatus which makes use of fast spectral correlation to gain insight into the behavior of a thin film of cyclohexane confined within attoliter volumes, with simultaneous measurement of film thickness and refractive index. With decreasing pore width, cyclohexane is found to undergo a drastic transition from a three-dimensional bulk fluid to a two-dimensional adsorbate with strikingly different properties. Long-range density fluctuations of unexpected magnitude are observed.

Several experimental techniques, such as the surface forces apparatus (SFA) (1), differential scanning calorimetry (DSC) (2), neutron diffraction (3), or nuclear magnetic resonance (4) in porous materials, are capable of measuring properties of a fluid under molecular-level confinement. Here, an extended surface forces apparatus (eSFA) (5) has been used, comprising two thin (2 to 5 m) mica sheets, which form a well-defined, single-slit pore of controllable width (i.e., surface separation), D (Fig. 1). The same mica sheets also form an optical interferometer using a 55-nm, silver-mirror coating on their reverse sides. The interferometer is immersed in a fluid, such as cyclohexane, and the fluid is increasingly confined between the surfaces by reducing D.

Cyclohexane becomes the central optical layer of a multi-layer interferometer, which selectively filters (infrared-free) white light in transmission. When the optical distance, D = n (D = film thickness, n = refractive index), of the thin film varies, the condition for constructive interference is altered, and the wavelength shift of interference fringes with different chromatic orders can be measured. Odd- and even-ordered fringes shift differently as a function of n, an effect that allows D and n to be independently determined. As one would expect, the measurement of n is more difficult for thinner films (5–7). Our measurement errors are 5% for distance and 3% for refractive index. Detailed error calculations are available (5). This represents a 10–to 30-fold improvement over conventional SFA techniques (6–12). Using imaging optics, transmitted light from a spot of lateral diameter 5 ¥ 10 m can be analyzed. For a very thin slit