Abstract—Signal processing is one of the most important features of electric circuits. It is the basis for electronics, sensing, triggering and acting to transform the physical principles of electricity into technology. This experiment will allow you to grasp some fundamental concepts on simple circuits and AC current by implementing a set of filters and amplifiers to produce a 2-band sound equalizer.

III. Sound Equalizer

Jorge D Morales

![Fig. 1. Schematic View](image)

Opamps were developed in the late 30’s to as telecommunications was barely arising. The signal on phone landlines was weak, and needed constant re-amplification. The solutions given at that time made the signal get rapidly distorted or still faint. Harry Black, came up with the first opamp design, with the following idea: the amplifier creates an excess of amplification, then sends back to the circuit some part of it (called a feedback) in a way that makes the circuit gain dependent on the feedback circuit rather than the amplifier gain. In this way the circuit gain depends on the passive feedback instead of the active amplifier [1]. Opamps produce control systems which are self-regulating through the feedback impulse, which makes them fairly stable and simple to use. The actual design of modern day opamps is achieved via transistors, diodes and such elements, but for the purposes of this procedure will not be taken into account. Instead the opamp will be tratred as a primary element, and its properties will be described to the necessary extent.

II. Implementation

A. RLC Circuits

Recall that resistor-inductor-capacitor circuits are oscillatory; they form a harmonic oscillator for current and produce resonance at certain frequencies. The nature of such oscillations may be inferred from the properties of the resistor, inductor, and capacitor.

The inductor and capacitor alone, produce resonance for certain frequencies. Both elements have an impedance (resistance analogous) that oscillates along with the circuit, so frequencies occurring when the impedance is at a minimum produce resonance. The rest of them suffer distortion, although some modes may coincide with harmonics of the original 'natural' frequency.

The resistor simply acts as a damper. That is, it impedes the signal to propagate freely, so that it decays. Resistors are power dissipaters, so they get rid of the energy passing through the AC wave.

Recall the general equation for an RLC circuit, constructed from Kirchhoff’s voltage law:

$$\frac{d^2i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{1}{LC}i(t) = 0$$ (1)
Which may be rewritten with the following factors:

\[
\frac{d^2i(t)}{dt^2} + 2\alpha \frac{di(t)}{dt} + \omega_0^2 i(t) = 0 \tag{2}
\]

Where \(\alpha\) is the attenuation factor or neper frequency (coupled to the resistor), and \(\omega_0\) is the angular frequency related to the natural frequency of the system. The solution to this equation is of the form \(\sim e^{st}\). Where \(s\) is a complex variable precisely dependant on the \(\alpha\) and \(\omega_0\) variables.

\[
s^2 + 2\alpha + \omega_0^2 = 0 \tag{3}
\]

The equation represents an oscillation that is attenuated for frequencies outside of the natural frequency and its modes. Furthermore, the Laplace Domain under this variable provides an easy approach to certain characteristics, which will be described afterwards.

### B. Damping Factor

The solutions for the \(s\) variable produce two interesting scenarios.

\[
s = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \tag{4}
\]

Where the turning point is given by the argument of the square root. A factor \(\zeta\) is defined as the ratio of \(\alpha\) over \(\omega_0\). Which contain the information for such scenarios. \(\zeta\) is named the damping factor.

Different kinds of systems may be defined through this factor:

- \(\zeta > 1\) Overdamped. The system returns in an exponential decay to its equilibrium without oscillation, the greater the slower it takes.
- \(\zeta = 1\) Critically Damped. The system returns to equilibrium as quickly as possible, without oscillating.
- \(0 < \zeta < 1\) Underdamped. The system oscillates and gradually returns to the equilibrium state.
- \(\zeta = 0\) Undamped. Harmonic oscillation.

Even more, an equivalent way of describing the system is through the Q factor (Quality Factor). In this case the ratio is \(\omega_0/2\alpha\), which allows to draw the same conclusions but instead with limits of \(1/2\), it also allows to understand the band response of different frequencies.

- \(Q = \inf\) Undamped. The frequency band is narrowed until only the natural frequency moves in harmonic oscillation.
- \(Q > 1/2\) Underdamped. Oscillation at some frequencies near \(\omega_0\) that decay gradually. The lower, the broader.
- \(Q = 1/2\) Critically Damped.
- \(Q < 1/2\) Overdamped.

### C. Laplace Domain

Laplace Transformations may be applied to linear systems, to represent the input, output signal. The transformation is applied to the differential equation, to change the second order dependence, to a linear dependence e.g.:

\[
Y(s) = H(s)X(s) \tag{5}
\]

Then, \(H(s)\) is the transfer function that linearly relates the input to the output:

\[
H(s) = \frac{Y(s)}{X(s)} = \frac{\mathcal{L}(g(t))}{\mathcal{L}(x(t))} \tag{6}
\]

In the case of a simple RLC circuit, the transformation may be applied (or simply revising the net impedance of the circuit and setting up Kirchhoff’s Law). The transformation in this case yields:

\[
V(s) = I(s) \left(R + sL + \frac{1}{sC}\right) \tag{7}
\]

If we identify \(I(s)\) to \(Y(s)\) and \(V(s)\) with \(X(s)\) then we can define the transfer function for our circuit to be:

\[
H(s) = \frac{s}{L \left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right)} \tag{8}
\]

The importance of this transformation is that we don’t really have to derive the solution of the particular differential equation but rather, after identifying its nature we can infer the characteristics and predict the values for the output.

### D. Opamp

As mentioned in the introduction, the internal configuration of an operational amplifier and its full understanding will not be covered in this project, but instead we will understand its operational features, and apply those rules to our known Kirchhoff Law to produce a high quality filter. Filters are in nature governed by RLC circuits but if an opamp is not included the response of the filter is more drastic and more biased by the filter resonance with the input signal, hence creating noise, especially with a signal that is not pure, but a rather rapidly variable and oscillating intensity (like music).

Opamps consist of three active terminals (the ones which will actually carry the signal and modify it): \(v_+, v_-, v_{cc}\). And two passive ends which are simply the opamp’s voltage source: \(v_{cc+}, v_{cc-}\). See Figure 2. Opamps require very little current to operate, nonetheless they do require a feeding voltage.

The simplest opamp configuration, without a feedback, just receiving a signal in the \(v_+\) input. Grounded through a
resistor in the $v_-$, and sending the output without feedback, in principle merely amplifies the voltage difference of $v_+$ and $v_-$. It amplifies in a way proportional to the Amplifier’s Open-Loop Gain. This is a specification particular of each device, and is commonly in the order of $10^4$. As mentioned, the purpose of the opamp is to work with a feedback, since it is such a feature that makes it special. The closed-loop configuration on the other hand (Figure ??) reduces dramatically the output gain, compensating the open-loop gain.

When working with a signal (generally with a feedback configuration) the opamp keeps the input terminals with no voltage difference ($v_+ = v_-)$, as the output terminal compensates for that. This also requires that the input terminals draw no current.

These simple set of rules turn the opamp into a black-box which merely follows the following rule:

$$v_+ = v_- = v_{out}$$  \hspace{1cm} (9)

And by applying Kirchhoff’s Law with these conditions, and obtaining the transfer function of $v_{out}/v_{in}$, and recognizing the terms in such Laplace domain, allow us to design a specific filter for specific frequencies and conditions.

The opamp, is also used with a final variation of the feedback terminal, that is setting up a voltage divider to make a constant ‘inner gain’. Such a configuration may be seen in Figure 3. And the output (hence a factor multiplying the original transfer function is added.

By following Kirchhoff’s Law,

$$v_{out} = v_- + IR_a$$  \hspace{1cm} (10)

where we know that I is equal to the current going only from $v_-$ to ground, passing through $R_g$, because the impedance between $v_+$ and $v_-$ should be so large that no current flows through the opamp. Then, since $R_g$ and $R_a$ are in an approximation to a series circuit, the current flowing through them is the same, thus $I = v_- / R_g$. We have, substituting back, and recognizing that the input voltages are equal:

$$v_{out} = v_+ + v_+ R_a R_g = v_+ \left(1 + \frac{R_a}{R_g}\right)$$  \hspace{1cm} (11)

Then the factor $G$, multiplies the transfer function $H$ whenever such a configuration is added. Making it simple to design a filter with no inner-gain, and finally multiplying such factor to the transfer function (within some corrections due to the circuit dependence):

$$G = \frac{v_{out}}{v_+} = 1 + \frac{R_a}{R_g}$$  \hspace{1cm} (12)

E. 2 Band Signal Split

1) Sound Considerations - Music: The purpose of this project is to apply a configuration of RC circuits with an opamp to divide the signal into two channels, a low pass and a high pass. Then control the gain of the amplification of each channel independently to allow to choose the tone of the signal, and finally add back the signals together.

It is required to notice that the human ear is capable of listening frequencies from 20Hz to 20kHz approximately. And that the pitch of notes has greater frequency steps in the lower frequencies. So in this project we will require to split the filter’s bandwidth in two sectors, below 1.8kHz, and above 2kHz. Leaving a 200Hz safe band gap between them. This gap is set to allow for underdamped frequencies of both filters to overlap without increasing strongly their amplitude.
2) Sallen-Key Topology: In this case, the filters have the following topology (Figure 4). The only difference between setting a highpass or a lowpass filter is the in the order of accommodating the capacitors and resistors of the active components. The schematic view shows impedances which can be substituted with the equivalent impedance of such devices.

At $V_z$ we have the following equation:

$$V_{in} - V_z = \frac{V_z - V_+}{Z_1} + \frac{V_z - V_{out}}{Z_2}$$

And given the opamp functionality, it becomes:

$$V_{in} - V_z = \frac{V_z - V_{out}}{Z_1} + \frac{Z_2}{V_4}$$

Furthermore, at $v_+$ and replacing $v_+$ with $v_{out}$:

$$V_z - V_{out} = \frac{V_{out}}{Z_3}$$

Finally, we obtain the equation:

$$V_z = V_{out} \left( 1 + \frac{Z_2}{Z_3} \right)$$

Which yields at last:

$$\frac{V_{out}}{V_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_1(Z_1 + Z_2) + Z_3 Z_4} \equiv H(s)$$

Which is in general the transfer function for our filter.

3) Low Pass Filter: The filter has a structure given by Figure 5. In this case the variables for $H(s)$ take the following form.

$$Z_1 = R_1 \quad Z_2 = R_2 \quad Z_3 = \frac{1}{sC_1} \quad Z_4 = \frac{1}{sC_2}$$

Furthermore we will define our filter to be in a near critically damped state (i.e. $Q=1/2$). And since there is no restriction we will allow the resistors and capacitors be equivalent to an integer proportionality of each other ($C_1 = nC_2$, $R_1 = mR_2$).

We can recognize the following expression for the transfer function (after substitution of previous expression of $H(s)$):

$$H(s) = \frac{1}{s^2 + \frac{R_1 + R_2}{C_1 R_1 R_2} s + \frac{1}{C_1 C_2 R_1 R_2}}$$

With this expression we can easily identify the natural frequency $\omega_0$, the quality factor $Q$, and the attenuation parameter $\alpha$. Which are the essential equations for our filter.

We realize that the system is overdetermined, so if we choose the natural frequency, and the quality factor, we can still impose the proportionality conditions mentioned above.

$$\omega_0^2 = \frac{1}{C_1 C_2 R_1 R_2}$$

$$2\alpha = \frac{R_1 + R_2}{C_2 R_1 R_2}$$

$$Q = \frac{\omega_0}{2\alpha}$$

Imposing such conditions over the quality factor, we obtain:

$$Q = \frac{\sqrt{mn}}{1 + m} = 1/2$$

Where we can easily identify that it is met when $m=n=1$.

Finally, we have come up with simple expressions (Letting $R_1 = R_2 = R$, and $C_1 = C_2 = C'$):

$$\omega_0 = \frac{1}{RC}$$

Which allows us to determine the value of one of the components, after fixing a natural frequency, (and recalling that $\omega_0 = 2\pi f_0$) and $f$ is the oscillation frequency (not angular frequency). Where in this case we want to set this filter to 1800Hz. And we are provided with capacitors of
Finally, without detailed calculations, we want to have an inner gain of $G=1.5$ (equation 12). And we will set $R_g = R$. So that $R_a$ is also to be calculated. Hence, $R$ and $R_a$, are the only components missing to build the low pass filter. The reason for choosing such a value of $G$ is because it normalizes the amplifier gain at $f_0$ to 1.

4) High Pass Filter: By following the exact same procedure, but with an inverted application of $R$ and $C$ (see Figure 6) we are able to determine the values of the resistors $R$ and $R_a$. We will set equally an inner gain of 1.5, a quality factor of 1/2 but in this case the frequency cut will be set at 2000Hz, and the Capacitor at $C=4.7\text{nF}$.

5) Inverting Amplifier: The following step is to create the amplifier of the signal. This configuration yields an inverted polarity output. Hence, the voltage will be inverted. In some systems that creates an issue, because some final components might depend on the original polarity. This is the case because speakers usually compensate the oscillation of their ’piston’ through particular designs which tend to involve damping factors, so that the positive and negative movements are compensated differently. The solution to this issue is to re-invert the signal through another device. We need one of these devices per frequency band.

An inverting amplifier has the structure shown in Figure 7.

In this case, after applying Kirchhoff’s Laws, we obtain:

\[ v_{\text{out}} = -R_2 \left( \frac{v_{\text{in}}}{R_1} \right) \] (25)

And in this case, $R_g$ is set to be equal to the parallel equivalent of $R_1$ and $R_2$. Sometimes it is not needed but in general it is used to reduce the offset bias current leakage, from the inputs.

Finally we may recall that the amplification is simply the parameter $\alpha = v_{\text{out}}/v_{\text{in}}$. In this case it is evidently negative, but we would like also to make it variable. We will chose to have a variation from -0.3 to -2.2.

To achieve this task we must set $R_1$ as a variable resistor. Such devices are also called potentiometers, a common value of this has a maximum resistance of $10\text{k}\Omega$, and a minimum value that will be assumed to be negligible in the scale. But applying a potentiometer is not just enough. The operational amplifier will not work with no impedance before the negative input. And a oscillation of $10\text{k}\Omega$ might be too much. So we must set an arrangement that begins with a minimum resistance, say $R_{11} = 1\text{k}\Omega$.

The minimum resistance $R_1$ will be $R_{11}$, and we want at this moment the maximum amplification, -2.2. With this, we can determine $R_2$ through equation 25.

Now, that we fixed $R_2$, and knowing that the minimum amplification must be -0.3 we need to determine $R_{1\text{MAX}}$. And set the simplest series/parallel combination possible, including the potentiometer to swipe through those values.

6) Summation Amplifier: Finally, we need to set up a configuration to add up the incoming signal from both amplifiers. This will enable the full spectrum of sound to be restored, but enhanced as the potentiometers are adjusted. In this case, the summation amplifier works similarly to the inverting amplifier. In fact, it inverts the signal back to its original polarity and the speakers are safe from this point.

The structure of this summation amplifier is as follows is on Figure 8. And it is practically the same structure as the inverter, with different incoming currents with their appropriate resistors. In this case we will only apply two, but more channels may be arbitrarily added in the same fashion.

Try to come up with the equation that yields this amplifier’s output. And calculate the amplification parameter of the summation amplifier shown in Figure 8.

Indeed a configuration combining the potentiometer and
The summation amplifier would provide a one step solution to the problem of adding and changing the amplitude of the channels, but some factors suggest that these independent devices are a better solution. (Try to think why).

The system is now integrated and should be appropriately connected. You should ground the system to the same ground line (coming from the signal source). In audio, it is the connector that is furthest from the tip (the third one), the other two are left and right. By now you might also have already guessed that this circuit is only mono (that is only left or right sound lines). The input signal is precisely one of these lines (left or right work equally), and naturally each component works in series. Refer to the Appendix Figure for a complete diagram of the circuit.

Calculate the total amplification of the signal before the summator, and after the summator (maximum and minimum values). And compare with your measurements. The average output signal of audio devices (for headphones) is 1.3V. What values do you expect to observe in the end, and what do you measure?

Recall that the opamps are delicate components, especially with the feeding voltage, assure that the appropriate polarity and intensity is set before feeding them, otherwise they burn up (no harm it just won’t work again).

III. Pre-Lab Procedure
Before proceeding to actually build our system we need to complete the circuit design. That is:

- Calculate remaining components of the Filters (R values, given the Capacitor’s values and the frequency cut).
- Calculate and propose the resistor design and values for the equivalent $R_1$, accordingly to the specifications of the inverting amplifier. (Minimum and maximum resistance and the resistor arrangement to obtain such resistance with the potentiometer.

- Calculate the amplification on each step, to obtain the final amplification factor.
- Obtain the equation that governs the summation amplifier.
- It is recommended that once you have all the calculations done you have a diagram of the circuit filled with all such parameters included, which is very helpful to map the circuit down to the board.

IV. Procedure
Now, knowing all the values and characteristics of the circuit we can actually proceed to build it and finally test it.

In this lab you will be using protoboards, the protoboard will serve as your building board for the circuit. See Figure ?? for reference. Wires run in the vertical direction in the center of the protoboard, and in the horizontal direction at the top as shown in this diagram, as you can see A through B are connected and F through J are also connected. Connecting and arranging the circuit simply consists in setting up the columns in the protoboard as independent nodes, and connecting the required ends (of resistors, capacitors or the opamp terminals) belonging to the same node. Note that the wires that run horizontally through the top and bottom of the protoboard have a break right at the center of the protoboard where the middle screw is that holds the protoboard in place. In order to compensate for this a wire must be run from the first section of this row to the second section so that the voltage runs through the entire row, this will be helpful for connecting different components with the same characteristics.

The opamp must be connected so that half the opamp is plugged into the E row and the other half is plugged into the F row (due to the actual shape of the opamps). This sets each terminal of the opamp in a different node, or column, otherwise they would be connected to the same node. To decide the specific best structure, refer to the technical data sheet provided online by the manufacturer. In this lab we will be using a JFET input opamp. Which is a common
device used for the kind of sensitivity, distortion and voltage parameters we need (i.e. Texas Instruments TL082, or ST UA741), although a general purpose operational amplifier works, as long as it is sensitive enough, and the input admits negative impulse (i.e. at least $\pm 3V$).

Figure ?? and 11 provide the conceptual view of an opamp (ST UA741), and the map for connecting the pins. Taken from the manufacturer’s datasheet [3].

The opamps work with an input voltage in the $30V$ range, that is the $v_{cc}$ inputs of the opamp (which naturally don’t act on the signal circuit, but just on the opamp feed) are to be provided by a DC source, set up on the manufacturer’s specified voltage difference. Warning, this is the most delicate part, connecting the feed voltage improperly or inversely will burn the opamp and waste it. So before turning the voltage source on, make sure to verify the feed is set up properly. The DC supplier works as a battery with a predefined voltage if the voltage is not enough, connect several in series, to add the voltage, they restrict due to safety issues the current output, but the opamps are supposed to function with negligible current.

We will also be using a Frequency generator see Figure 12, the frequency generator can be connected to the circuit, it outputs different frequencies and functions which can be seen if the circuit is then attached to a oscilloscope, which is another item we will be using to test the circuit. Use the function generator, to observe the response of the filter to the different frequencies. Set the frequency in different ranges, and then proceed to compare the signal amplitude before and after the components we are building. Plug the Hi Lo output terminals to the ground and input terminals of our design. (Don’t confuse the ground with the negative feed $v_{cc}$ they are independent, the feed is not counted in the circuit’s diagram, but only in the connection of the opamps).

A. Test with Oscilloscope

Then we shall use the oscilloscope to actually see the signal. The oscilloscope basically takes voltage amplitude readings which are then displayed on a graph (V vs time). See Figures 13, and 14 for a reference of two kinds of oscilloscopes (digital and analogical). An oscilloscope connector is plugged in through the coaxial cable, and since it is used to read the signal, should be grounded to the circuit’s ground, and the active input positioned at the node in which the oscillation is read. It is recommended to use a two channel oscilloscope to compare both signals at the same time. (input and output signals). That way you can verify what the circuit is doing and test every part of the circuit while you have an input, and find the non working or malfunctioning elements if they exist.

B. Test with Music

Once the filters are tested with the function generator, music may be connected. If done properly there is no feedback on the input terminals so the connected device is safe, actually this is protected by the opamp configuration. Connect music and observe the variation of the signal with the oscilloscope. The last part needed is some speakers. Once
the system is proved to be working without inverting the polarity, and in the amplitude range desired, you may connect the speakers, in the same way as the oscilloscope, grounding to the circuit’s ground, and transferring the output to one of the left-right channels.

1) Step Enumeration:

1) This lab will be done in teams, each creating a small set of the components.
2) Assemble the filter. Make sure to connect each wire to the correct location on the op-amp. Test with the function generator and the oscilloscope. Remember to ground the circuit to the same node, but that the negative input voltage $v_{cc}$ of the opamp is not. Measure the amplification, at different frequencies, including the natural frequency.
3) Build the inverting amplifier, using the potentiometer located in the breadboard, test this component with different frequencies and measure the amplification.
4) Connect the output of the filter to the input of the inverting amplifier. Also connect grounds and input voltages. Measure the combined amplification.
5) Match with another team compensating for the bandwidth of your device.
6) Next construct the summation amplifier on its own protoboard, (remember this gadget should revert the polarity) as it combines and inverts the signal back to the correct polarity, make sure that is happening. Measure the amplification factor of this device independently, and then the amplification of the whole system (at different frequencies).
7) Now that the equalizer is finished we can begin testing it with music.
8) You must then connect an auxiliary cord to the input and connect both filters to create a common node. And naturally, grounding the system to the chord’s ground line.
9) Finally attach your equalizer to the speakers and you can begin playing music.
10) Vary the resistance in both the high and low pass filter and observe the results.

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REFERENCES

2 BAND EQUALIZER
1200 HZ - 2000 HZ

High Pass Filter

Low Pass Filter

Inverting Amplifier

Summation Amplifier