II. Millikan Oil Drop Experiment

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Fig. 1. Millikan Cross Section

Abstract—The experiment was devised by American physicist Robert A. Millikan in 1909. Millikan wanted to determine whether electric charges occurred in discrete units which were integral multiples of some smallest charge $e$ (which turns out to be carried by an electron or proton), and to measure that charge. The present apparatus uses nylon beads rather than the oil droplets used by Millikan, but in most other respects is the same.

I. INTRODUCTION

The apparatus is shown in cross-section in Figure 1. An atomizer is used to spray an aerosol suspension of tiny spherical nylon beads into an upper chamber. Most of the beads receive a small net charge from friction as they emerge from the nozzle of the atomizer. (Note that this is analogous to the charging of the belt on a Van de Graaf generator). Some of the beads fall through a hole in the center of a metal plate (upper electrode), so that they begin to fall through the air in the lower chamber.

Since the spheres are far too small to be seen by the naked eye, an illuminator and a microscope are used to observe them (Figure 2). Each sphere is seen as a small point of light. When the potential difference between the upper and lower electrodes is zero, the sphere falls slowly and steadily with a terminal velocity, corresponding to the equilibrium between the downward force of gravity and the upward viscous force of the air. (There also is an upward buoyant force equal to the weight of the air displaced by the droplet, but it will be neglected because it is less than one-thousandth the weight of the droplet). The situation is similar to that of a parachutist falling through the air. When a potential difference is applied to the electrodes, an additional electric force acts and the terminal velocity is changed (if the droplet is charged).

As will be seen below, if the density of nylon and the radius of the droplet are known, and the terminal velocity is measured, with and without an applied electric field $E$, the force of the electric field on the bead can be determined, and from this the charge $q$ on the bead can be determined. A series of terminal velocity measurements will indicate groups of $q$ values that are seen to multiples of a smallest value $e$, which is hypothesized to be the fundamental electronic charge. The accepted value of $e$ is $1.6 \times 10^{-19} C$.

A. Levitation: equilibrium between electric and gravitational forces

First consider a droplet or radius $R$ that is suspended at constant vertical position by an exact balance between upward electric force and a downward gravitational force. Figure 3.

Now consider what happens to a solid sphere of radius $R$, if only a modest number of surface atoms have net charge. Assume that the surface charge density is $\sigma$. Then the potential at the surface is:

$$V = \frac{(\sigma 4 \pi R^2)}{4 \pi \epsilon_0 R} = \frac{\sigma R}{\epsilon_0}$$  \hspace{1cm} (1)

The electric field at the surface of the sphere is:

$$E = \frac{(\sigma 4 \pi R^2)}{4 \pi \epsilon_0 R^2} = \frac{\sigma}{\epsilon_0}$$  \hspace{1cm} (2)

The significance of this electric field is that the limit to the amount of charge that can be developed on an object arises from its ability to ionize atoms of gas near the surface. The ionization process is a function of electric field. In dry
Fig. 3. Levitation

air, for example, the maximum surface breakdown field is $E_b \simeq 10^6 \text{V}$.

The maximum electric force that can be exerted on the sphere by an external electric field $E_{\text{ext}}$ is then:

$$F_e = (\sigma 4\pi R^2) \cdot E_{\text{ext}} = \varepsilon_0 E_b 4\pi R^2$$

Now the gravitational force on the same sphere accumulates from the force on all the matter within the sphere, so

$$F_g = \frac{\rho \cdot 4\pi R^3 \cdot g}{m}$$

Since the gravitational force increases as $R^3$ while the maximum electric force increases like $R^2$, we need to work with very small spheres in order to attain an equilibrium between these two forces.

If we knew the mass of the droplet, we could solve directly for the charge on the droplet. In practice, we will be using tiny plastic beads (instead of the oil droplets used by Millikan), but the size (and mass) of the beads varies significantly within the sample, so that for a precise measurement we would need to measure the mass of each bead. We can actually do this within the Millikan experiment by observing the beads, not in equilibrium levitation, but in free fall in which the electric and gravitational forces are not quite in balance.

B. Free fall

Next consider a sphere of mass $m$, charge $q$, and radius $r$ falling under the influence of gravity only (acceleration of gravity = $g$). As noted above, the speed of the sphere rapidly increases until a constant terminal velocity $v_0$ is reached. This happens when the weight $mg$ of the sphere (minus the buoyant force) is exactly equal (and opposite) to the frictional force exerted by the air. This problem was first considered by Sir George Stokes, who derived an expression for the viscous force $F_f$ that acts upon a particle of radius $r$ as it passes through a fluid with velocity $v$ [1]:

$$F_f = 6\pi \eta \nu v$$

The quantity $\eta = 1.85E^{-5} \text{kgs/m}$ is the coefficient of viscosity of air at room temperature. Thus in the absence of electric field:

$$E = 0 \ : \ mg = 6\pi \eta \nu v_0$$

Where $\nu_0$ is a positive number. Note that the radius $r$ of the sphere can be determined from this measurement of $v_0$ in the absence of electric field. The mass of the bead is $m = \frac{4}{3} \pi r^3 \rho$. Note: you will need to find a value for the density of nylon, either in a library source or the Internet. We can therefore solve Eq. 2 for the radius:

$$r = 3 \left( \frac{\eta m \nu}{2 \rho g} \right)^{1/3}$$

Eq. 3 is Stokes Law, relating the radius of a spherical body to its terminal velocity as it falls in a viscous medium. (Where $F_f = mg$)

Stokes Law, however, becomes incorrect when the velocity of fall of the droplets is less than $0.1 \text{ cm/s}$. Droplets having this and smaller velocities have radii $< 2 \mu\text{m}$, comparable to the mean free path of air molecules. The derivation of Stokes Law assumes that the sphere radius is larger than this mean free path. Since the velocities of the droplets used in this experiment will be in the range of 0.01 to 0.001 cm/s, the coefficient of viscosity must be multiplied by a correction factor [2]:

$$\eta_{\text{eff}} = \frac{\eta}{1 + \frac{b}{p r}}$$

where $b = 6.17E^{-4} \text{mmHgcm}$ is a constant, $p = 760 \text{mmHg}$ is the atmospheric pressure, and $r$ is the radius of the drop as calculated by the uncorrected form of Stokes Law. Substituting $\eta_{\text{eff}}$ from equation (8) into equation (7) and solving for the radius $r$ gives:

$$r = \sqrt{\left( \frac{b}{2p} \right)^2 + \frac{9\nu_0 \eta}{2g\rho} - \frac{b}{2p}}$$
C. Electric and gravitational forces in equilibrium

Now suppose that you apply an electric field \( E \), in a direction such as to cause an upward force on the bead sufficient to cause it to move upward with a terminal velocity \( v_+ \). Since the bead is now moving upward, the viscous force acts downward (Figure 4). The equation of equilibrium is:

\[
qE = mg + 6\pi \eta r \nu_+ \tag{10}
\]

Now suppose that the sign of the \( E \)-field is reversed. The viscous force now acts upward, Figure 5.

\[
qE + mg = 6\pi \eta r \nu_- \tag{11}
\]

where \( \nu_- \) is the new (downward) terminal velocity. If Eq. 6 is added to Eq. 7, we eliminate the gravity term and obtain:

\[
qE = 3\pi \eta r (\nu_+ + \nu_-) \tag{12}
\]

But, since \( E = V/d \), where \( V \) is the potential difference between the plates and \( d \) is the plate separation, this reduces to:

\[
q = \frac{3\pi \eta d r (\nu_+ + \nu_-)}{V} \tag{13}
\]

In summary, a measurement of \( v_0 \) and use of Eq. 5 leads to a determination of \( r \), and further measurements of \( v_+ \) and \( v_- \), enable us to determine the charge \( q \) on the bead.

II. Setting up the Equipment

NOTE! The power should be turned off when any adjustments of the apparatus are being made.

1) Install the atomizer ring. This is done by unscrewing the electrode housing set screws, removing the upper electrode housing plate, placing the atomizer ring with openings for the illuminator and microscope oriented properly, replacing the upper electrode housing plate, and retightening the set screws. Insert the loose end of the narrow tube into the storage bottle.

2) Make sure that the color-coded D.C. voltage leads are plugged into the corresponding color-coded terminals on the power supply.

3) Set the 3-way polarity switch to its mid-position, which puts no voltage on the electrodes. Turn the on/off switch to on. The illuminating lamp should light.

4) Adjust the microscope by turning the focus-adjusting knob until an approximate mid-position is established. The scale divisions should be distinguishable from the background.

5) Set the electrode potential to 300 V using the voltage adjustment knob, but keep the polarity switch in its mid-position.

6) In order to use the atomizer, first loosen the set screws of the electrode housing so as to permit air to escape. Then inject nylon spheres into the chamber with the atomizer by covering the air hole of the spray bulb with a finger and squeezing the bulb. The hole must be covered for successful injection.

Spraying is usually difficult the first few times. It is recommended that you make several test sprays before attempting to make measurements. While spraying nylon beads into the apparatus, carefully look for the dots of light in the microscope. Adjust the focus of the microscope until you can see them clearly. Note that beads continue entering the region between the plates for several seconds after the spraying has stopped. Practice until you are comfortable with the microscope.
A. Procedure

Adding one electron.

A net electric charge by either stripping off one electron or of the atoms on the surface layer of an object can be given charge but no net electric charge. In general only some fraction so almost all the matter in an object has net gravitational charges of electrons, but like charges repel one another. That the positive charges in nuclear matter attract the negative electric charges in matter have both + and sign, meaning that gravitation is always attractive between objects. But fundamental difference between these two forces of nature.

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With the equipment at hand your team will need to work in close coordination. One person is needed to observe the droplet and operate a stopwatch. A second person will record the stopwatch readings, the polarity of the E-field, and the voltage reading. A third one will operate the atomizer and control the voltage. For good results it is essential that readings for \( E = 0 \), +, and − be made on the same bead, which means that the observer must keep the bead continuously in view.

III. Problem 1

LEVITATION

You are going to use the electric force to defy gravity. In order to accomplish this feat, you take into consideration a fundamental difference between these two forces of nature. The gravitational charge (i.e. mass) of all matter has a single sign, and gravitation is always attractive between objects. But the electric charges in matter have both + and sign, meaning that the positive charges in nuclear matter attract the negative charges of electrons, but like charges repel one another.

This property of opposite charges has the consequence that atoms form electrically neutral building blocks of matter, and so almost all the matter in an object has net gravitational charge but no net electric charge. In general only some fraction of the atoms on the surface layer of an object can be given a net electric charge by either stripping off one electron or adding one electron.

A. Procedure

You will be using nylon beads of 4\( \mu \text{m} \) diameter. You should find the best value for the mass density of nylon, so that you can estimate the gravitational force.

The spheres are given a small (random) amount of electric charge as they are shot into the chamber through the atomizer nozzle. The rubbing of each sphere against the side walls of the nozzle and against one another is quite sufficient to give the spheres the amount of charged needed to be able to levitate them.

In this problem, you will levitate a sphere, and make an approximate measurement of its total charge by measuring the electric field needed for levitation.

Rotate the roles of operator, observer, and data logger during the experiment so that each partner observes the levitation. You should make a histogram of the charge measurements on successive spheres (at least 8), and see if your measurements can all be characterized as an integer times a fundamental unit of charge, hence verifying that electric charge is quantized in matter.

IV. Problem 2

MEASUREMENT OF RISING AND FALLING DROPS

Millikan did this classic experiment using droplets of oil sprayed from an atomizer. He was able to make quite small droplets (down to 1 \( \mu \text{m} \)) but the droplet sizes were not uniform. In order to make consistent measurements, he had to simultaneously measure the droplet size and the charge state. He devised the procedure described above, in which for each droplet one measures the velocity for free fall and the terminal velocity when an external field is applied to make the sphere rise or fall. You will follow his procedure to make such simultaneous measurements.

A. Procedure

1) Record the time to fall 1 mm for a single droplet under the three conditions: 0, +, and −. Calculate the corresponding velocities \( v_0 \), \( v_+ \), and \( v_- \), and then calculate the charge \( q \).
2) Repeat Step 1 for as many different droplets as you can.
3) Plot a histogram of the observed charge values. They should fall into equally spaced clusters, corresponding to multiples of a single fundamental charge \( e \).
4) Select a well-behaved droplet and measure \( v_+ \) for several different accelerating voltages \( V \). Make a plot of \( v_+ \) vs. \( V \). Interpret the slope and intercept of this plot. Would you have expected the slope and intercept to be the same if you had selected a different bead?
5) Prepare a histogram of the distribution of charge states on the beads that you observe. The friction charging mechanism is statistical in nature. The distribution of number of electrons on each bead should be a Poisson distribution. Find a reference on Poisson distributions,
calculate the mean charge of your distribution, and then calculate the Poisson distribution that corresponds to that mean. Compare to your histogram.

B. Cleaning up after the experiment

It is important that the atomizer ring and atomizer be cleaned after the measurements have been completed. Disassemble the ring and electrode structure by reversing Step 1 of the setup sequence. Clean the electrodes and ring by washing with water and drying with tissue. Clean the pump by immersing the end of the small tube in a beaker of clean water and pumping several times. Withdraw it from the beaker and pump until it stops spaying water. Restore the apparatus to the state in which you found it.

REFERENCES

