$U = qV$

Power is the rate of change of energy

$$P = \frac{dU}{dt} = \frac{dq}{dt}V$$

$P = IV$

Units of power:

$$\text{Power} = \left(\frac{\text{energy}}{\text{time}}\right)$$

$$1 \text{ joule/}\text{sec} = 1 \text{ watt}$$

$V = IR$: $P = I(IR)$

$P = I^2R$ (Joule’s Law)

$I = \frac{V}{R}$: $P = \left(\frac{V}{R}\right) V$

$P = \frac{V^2}{R}$
Example:

Light bulb - 100 W (120 V)

\[
I = \frac{P}{V} = \frac{100}{120} = 0.833 \text{ A}
\]

\[
R = \frac{V^2}{P} = \frac{120^2}{100} = 144 \Omega
\]

or

\[
R = \frac{V}{I} = \frac{120 \text{ V}}{0.883 \text{ A}} = 144 \Omega
\]
ac Circuits

\[ v(t) = V_0 \sin 2\pi ft \]

\[ i(t) = \frac{v(t)}{R} = \frac{V_0}{R} \sin 2\pi ft = I_0 \sin 2\pi ft \]
Average potential difference = 0
and
Average current = 0

However, the power dissipated in R is
\[ p(t) = i^2(t) R \]

**SQUARE** of the current

\[
I_0^2 \\
\frac{1}{2}I_0^2 \\
0 \\
\frac{1}{2}I_0^2 \\
I_0^2
\]

**MEAN** (average) of \( i^2(t) \) is \( \frac{1}{2}I_0^2 \)

**ROOT-MEAN-SQUARE** (rms) value of the current is

\[
I_{rms} = \sqrt{\frac{1}{2}I_0^2} = \frac{I_0}{\sqrt{2}}
\]
Similarly, the rms value of the voltage is

\[ V_{\text{rms}} = \frac{V_0}{\sqrt{2}} \]

The average power dissipated in \( R \) is

\[ \overline{P} = \overline{p(t)} = \overline{i^2(t)} \cdot R = \frac{1}{2} I_0^2 \cdot R \]

Thus,

\[ \overline{P} = I_{\text{rms}}^2 \cdot R \]

Using rms values, the ac equations have identical forms to those for dc currents and voltages:

\[ V_{\text{rms}} = I_{\text{rms}} \cdot R \]

\[ \overline{P} = I_{\text{rms}} \cdot V_{\text{rms}} \]

\[ \overline{P} = \frac{V_{\text{rms}}^2}{R} \]
Resistors in Series

\[ V = V_1 + V_2 + V_3 \]

\[ I R_s = I R_1 + I R_2 + I R_3 \]

\[ R_s = R_1 + R_2 + R_3 \]
Resistors in Parallel

\[ I = I_1 + I_2 + I_3 \]

\[ \frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \]

\[ \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]
Example:

\[ \frac{1}{R_P} = \frac{1}{12} + \frac{1}{6} = \frac{1}{12} + \frac{2}{12} = \frac{3}{12} \]

\[ R_P = \frac{12}{3} = 4 \, \Omega \]

\[ R_S = 4 \, \Omega + 5 \, \Omega = 9 \, \Omega \]
Circuit Symbols:

- cell
- battery
- capacitor
- capacitor (variable)
- resistor
- resistor (variable)
- fuse
- switch
- ground
Kirchoff’s Laws

I. (Conservation of Charge) -
   The algebraic sum of the currents entering any junction point in a circuit is zero.

II. (Conservation of Energy) -
   The algebraic sum of the changes in potential around any closed path in a circuit is zero.
Note when applying Kirchoff’s II law:

For a resistor there is always a potential drop in the direction of current flow

\[ V = I \times R \]

For a battery or cell the potential change is determined by the orientation of the battery and is independent of the current direction.
Demonstration of Kirchoff’s laws

\[ +1.5 - 0.5 - 1.0 = 0 \]
\[ +4.0 - 1.5 - 2.5 = 0 \]
\[ +2.0 - 2.0 - 4.0 = 0 \]

\[ +9.0 - 1.0 \times 9.0 = 0 \]
\[ +1.0 \times 9.0 - 9.0 = 0 \]

\[ -1.0 \times 9.0 + 2.0 \times 3.0 + 1.5 \times 2.0 = 0 \]
\[ I_1 - I_2 - I_3 = 0 \]
\[ 9 - 5 I_1 - 12 I_2 = 0 \]
\[ 12 I_2 - 6 I_3 = 0 \]
\[ I_2 = I_1 - I_3 \]

\[
9 - 5 I_1 - 12 (I_1 - I_3) = 0
\]

\[
12 (I_1 - I_3) - 6 I_3 = 0
\]

\[
9 - 17 I_1 + 12 I_3 = 0
\]

\[
12 I_1 - 18 I_3 = 0
\]

\[
I_1 = \frac{18}{12} \quad I_3 = \frac{3}{2} I_3
\]

\[
9 - 17 \left(\frac{3}{2} I_3\right) + 12 I_3 = 0
\]

\[
I_3 = \frac{2}{3} \quad A
\]

\[
I_1 = \frac{3}{2} \quad I_3 = 1 \quad A
\]

\[
I_2 = I_1 - I_3 = \frac{1}{3} \quad A
\]
\[
\frac{1}{R} = \frac{1}{12} + \frac{1}{6} = \frac{3}{12} ; \quad R = \frac{12}{3} = 4 \ \Omega
\]
\[ \begin{align*}
I_1 - I_2 - I_3 &= 0 \\
9 - 5I_1 - 12I_2 &= 0 \\
12I_2 + 18 - 6I_3 &= 0
\end{align*} \]

\[ I_2 = I_1 - I_3 \]

\[ \begin{align*}
9 - 5I_1 - 12(I_1 - I_3) &= 0 \\
12(I_1 - I_3) + 18 - 6I_3 &= 0
\end{align*} \]
\[ 9 - 17 I_1 + 12 I_3 = 0 \]
\[ + 18 + 12 I_1 - 18 I_3 = 0 \]

\[ 9 - 17 I_1 + 12 I_3 = 0 \]
\[ + 12 + 8 I_1 - 12 I_3 = 0 \]

\[ 21 - 9 I_1 = 0 \]

\[ I_1 = \frac{21}{9} \text{ A} \]

\[ + 12 + 8 \times \frac{21}{9} - 12 I_3 = 0 \]

\[ I_3 = \frac{23}{9} \text{ A} \]

\[ I_2 = I_1 - I_3 = \frac{21}{9} - \frac{23}{9} \]

\[ I_2 = -\frac{2}{9} \text{ A} \]
Terminal Voltage of a Cell

\[ E - Ir - IR = 0 \]

\[ I = \frac{E}{r + R} \]

\[ V_T = E - Ir = E \frac{R}{r + R} = E - \frac{Er}{r + R} \]
Note:

\[ R=0 \quad \Rightarrow \quad V_T=0 \quad \Rightarrow \quad I = \frac{E}{r} \]

**Short Circuit Current**

\[ r \rightarrow \infty \quad \Rightarrow \quad V_T \rightarrow 0 \]

**Dead Battery**

\[ R \rightarrow \infty \quad \Rightarrow \quad V_T \rightarrow E \]

**Open Circuit Voltage**
Discharging:

\[ V_T = E - Ir \]

Charging:

\[ V_T = E + Ir \]