Surface of conducting sphere:

\[ E_s = \frac{kQ}{R^2} \quad V_s = \frac{kQ}{R} \]

\[ V_s = R \cdot E_s \]

EXAMPLE:

1 µC on a metal sphere of radius \( R = 9 \) cm.

\[ V_s = \frac{9 \times 10^9 \times 1 \times 10^{-6}}{.09} = 100,000 \text{ V} \]

\[ E_s = \frac{9 \times 10^9 \times 1 \times 10^{-6}}{.09 \times .09} = 1.1 \times 10^6 \text{ V} \]

\[ = 1.1 \times 10^6 \text{ N/C} \]
At STP air breaks down (arcs) when the electric field reaches a value

\[ E_{\text{max}} \approx 3 \times 10^6 \text{ V/m} \]

This maximum field strength in an insulating material is called the

"Dielectric Breakdown Strength"

For a sphere of radius \( R \)

\[ E_s = \frac{V_s}{R} \]

Hence

\[ V_{\text{max}} = R E_{\text{max}} \]
Thus for air at STP

\[ V_{\text{max}} = 3 \times 10^6 R \]

where \( V_{\text{max}} \) is in volts and \( R \) is in meters.

Examples:

R=1 m \hspace{1cm} V_{\text{max}} = 800,000 V.

R = 10^{-6} m \hspace{1cm} V_{\text{max}} = 0.8 V

The latter is typical of a sharp needle.
For a sphere of Radius R at a fixed potential V,

\[ V_{\text{max}} = \frac{kQ}{R} = \frac{Q}{4\pi\varepsilon_0 R} \]

If the charge density on the surface of the sphere is \( \sigma \), then

\[ Q = 4\pi R^2 \sigma \]

and thus,

\[ V_s = \frac{4\pi R^2 \sigma}{4\pi \varepsilon_0 R} \]

or

\[ \sigma = \frac{\varepsilon_0 V_s}{R} \]

Since \( E_s = \frac{V_s}{R} \), we have

\[ \sigma = \varepsilon_0 E_s \]
Consider a conducting body. All points of it must be at the same potential.

\[ \sigma \text{ and } E \text{ are larger at places where the radius of curvature is smaller} \]
Parallel Plates

Electric Field Lines

Equipotential Surfaces

E is constant between the plates. \[ E = \frac{\Delta V}{\Delta s} \]

EXAMPLE:

\[ \Delta s = 1 \text{ cm} \]

\[ E = \frac{\Delta V}{\Delta s} = \frac{1.5}{.01} = 150 \text{ V/m} = 150 \text{ N/C} \]
The Electron-Volt (eV)

The electron-volt is a unit of energy.

\[ U = qV \]

\[ q \equiv e = 1.6 \times 10^{-19} \text{ C} \quad V = 1 \text{ Volt} \]

\[ 1 \text{ eV} = 1.6 \times 10^{-19} \text{ C} \times 1 \text{ Volt} \]

\[ = 1.6 \times 10^{-19} \text{ J} \]

\[ 1 \text{ keV} = 10^3 \text{ eV} \]

\[ 1 \text{ MeV} = 10^6 \text{ eV} \]

\[ 1 \text{ GeV} = 10^9 \text{ eV} \]
CAPACITANCE

It is the ability to store charge.

It depends only on geometry

\[ C = \frac{\text{amount of charge stored}}{\text{potential difference}} \]

\[ C = \frac{Q}{V} \]

Units: \( 1 \text{ Farad} = 1 \text{ C/V} \)

\( 1 \mu\text{F} = 10^{-6} \text{ Farad} \)

\( 1 \text{ pF} = 10^{-12} \text{ Farad} \)
EXAMPLE:

Capacitance of a conducting sphere whose radius is $R$.

\[
C = \frac{Q}{V} = \frac{Q}{kQ/R} = \frac{R}{k}
\]

Note that $C$ depends only on the geometry.
Parallel Plate Capacitor

\[ V = -\int_{a}^{b} \overrightarrow{E} \cdot d\overrightarrow{\ell} = Ed \]

\[ = \frac{\sigma}{\varepsilon_0}d = \frac{1}{\varepsilon_0} \frac{Q}{A}d \]

\[ C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d} \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2} \]

Note: \( C \) depends only on geometrical factors.
EXAMPLE:

\[ C = \frac{8.85 \times 10^{-12} \times 0.1 \times 0.1}{8.85 \times 10^{-3}} \]

\[ = 10^{-11} \text{ F} = 10 \text{ pF} \]

\[ Q = CV = 10^{-11} \times 1.5 = 1.5 \times 10^{-11} \text{ C} \]

\[ Q = 15 \text{ pC} \]
Consider a parallel plate capacitor:

\[ + + + + + + + + \]

\[ \hat{E} \]

\[ + + + + + + + + \]

\( C_0 \) is capacitance with vacuum between the plates

\[ + + + + + + + + \]

Dielectric Slab

\[ + + + + + + + + \]

\( C \) is capacitance with a dielectric slab between the plates

Definition: Dielectric constant \( \kappa \) is

\[ \kappa = \frac{C}{C_0} \geq 1 \]
Microscopic Model

Atom in Absence of Electric Field

Spherically symmetric charge distribution

Atom in Presence of Electric Field

E
Polarization

Model of a dielectric medium between plates of a charged capacitor
σ is the charge per unit area stored on the plates.

σ_i is the charge per unit area induced on the surfaces of the dielectric.
Vacuum between the plates:

\( V_0 \) is the potential difference

\( E_0 \) is the electric field

\[
\frac{V_0}{d} = E_0 = \frac{\sigma}{\varepsilon_0}
\]

Dielectric \( \kappa \) between the plates:

\( V \) is the potential difference

\( E \) is the electric field

\[
\frac{V}{d} = E = \frac{\sigma - \sigma_i}{\varepsilon_0}
\]

Thus

\[
\kappa = \frac{C}{C_0} = \frac{Q/V}{V_0} = \frac{V_0}{V} = \frac{E_0}{E} = \frac{\sigma}{\sigma - \sigma_i}
\]
Clearly, we always have
\[ V_0 \geq V \quad \text{and} \quad E_0 \geq E \]

Since
\[ \kappa = \frac{\sigma}{\sigma - \sigma_i} \]

we have
\[ \sigma_i = \sigma \left(1 - \frac{1}{\kappa}\right) \]

Also,
\[ E = \frac{E_0}{\kappa} = \frac{\sigma}{\kappa \varepsilon_0} \]

Defining
\[ \varepsilon = \kappa \varepsilon_0 \]

gives
\[ E = \frac{\sigma}{\varepsilon} \]
\[ C = \kappa C_0 = \kappa \varepsilon_0 \frac{A}{d} \]
\[ C = \varepsilon \frac{A}{d} \]