ELECTRIC FIELD at point \( p \) is the force that would be exerted on a unit charge placed at point \( p \). The Electric field exists at every point in space.
\[ \vec{F} = Q \vec{E} \]
\[ \vec{E} = \frac{\vec{F}}{Q} \]

\( \vec{E} \) is a vector. It has both **MAGNITUDE** and **DIRECTION**

The convention for the direction of \( \vec{E} \) at a point is the direction of the force on a + charge placed at the point.

**Units of \( E \):**

newtons/coulomb

N/C
What is \( \vec{E} \) at point \( p \) due to the charge \( Q \)?

Put a test charge \( q_t \) at point \( p \). The force on \( q_t \) due to \( Q \) is

\[
F = \frac{kQq_t}{r^2}
\]

and

\[
E = \frac{F}{q_t} = \frac{kQ}{r^2}
\]
Thus the magnitude of the electric field at point \( p \) due to the charge \( Q \) is

\[
E = \frac{kQ}{r^2}
\]

NOTE:

\( Q \) produces \( E \). The effect of \( E \) on another charge \( Q' \) is a force

\[
\vec{F} = Q'\vec{E}
\]

NOTE:

\[
k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9
\]
Example:

\[ Q = 2 \text{ C} \quad r = 3 \text{ m} \]

\[ +2 \text{ C} \]

\[ 3 \text{ m} \]

\[ p \]

\[ \vec{E} \]

We find:

\[ E = \frac{9 \times 10^9 \times 2}{9} = 2 \times 10^9 \text{ N/C} \]
Electric field at a point $P$ due to several point charges? Use the

**Principle of Superposition:**

Determine the electric field at the point $P$ due to each of the charges. Then, the net electric field at $P$ is the vector sum of these fields.
Example:

\[ E = \frac{9 \times 10^9 \times 2}{9} = 2 \times 10^9 \text{ N/C} \]

Each charge produces a field
Add these two fields vectorially

Components of $E$:

**Horizontal:** $E \sin 30^\circ$

\[
= 2 \times 10^9 \times \frac{1}{2} = 1 \times 10^9 \text{ N/C}
\]

**Vertical:** $E \cos 30^\circ$

\[
= 2 \times 10^9 \times \frac{\sqrt{3}}{2} = 1.73 \times 10^9 \text{ N/C}
\]

The components of the total field are:

$E_x = 1 \times 10^9 - 1 \times 10^9 = 0$

$E_y = 2 \times 1.73 \times 10^9 = 3.46 \times 10^9 \text{ N/C}$
ELECTRIC FIELDS

produced by

CONTINUOUS CHARGE DISTRIBUTIONS

\[ dE = k \frac{dQ}{r^2} \]

\[ \vec{E} = \int d\vec{E} \]

Example:

A charge \( Q \) is uniformly distributed on a thin ring of radius \( a \). What is the field on the axis?

The charge per unit length on the ring is

\[ \lambda = \frac{Q}{2\pi a} \]
\[ dE_x = \frac{dE \cos \theta}{dE \cos \theta} \]

\[ dl = r \left( \frac{Q}{2\pi a} \right) \frac{\cos \theta}{r^2} \int_0^{2\pi} dl = k \frac{Q \cos \theta}{r^2} = k \left( \frac{Q}{z^2} \right) \frac{z}{(z^2 + a^2)^{\frac{3}{2}}} \]

For \( z \gg a \)

\[ E = \frac{kQ}{z^2} \] (Field of point charge)
Example: A charge $Q$ is uniformly distributed on a thin disk of radius $R$. What is the field on the axis?

For the ring of charge,

$$dQ = 2\pi a \sigma da$$
and

\[
dE = \frac{k \, z \, dQ}{(z^2 + a^2)^{3/2}} = \frac{z \sigma \, a \, da}{2 \epsilon_0 \left(z^2 + a^2\right)^{3/2}}
\]

\[
E = \frac{z \sigma}{2 \epsilon_0} \int_0^R \frac{a \, da}{\left(z^2 + a^2\right)^{3/2}} = \frac{\sigma}{2 \epsilon_0} \left[ 1 - \frac{z}{\sqrt{z^2 + R^2}} \right]
\]

In the limit \(z \ll R\) (points very close to the disk), the field becomes

\[
E = \frac{\sigma}{2 \epsilon_0}
\]

and is independent of \(z\)!
Example:

What is the electric field at the origin due to a charge $Q$ uniformly distributed on a quarter circle?

\[
dE_x = dE \cos \theta \\
dE_y = dE \sin \theta \\
\lambda = \frac{Q}{\frac{1}{2} \pi R} = \frac{2Q}{\pi R} \\
dQ = \lambda R \, d\theta = \frac{2Q}{\pi} \, d\theta
\]
\[ dE = \frac{k \, dQ}{R^2} = \frac{2k \, Q}{\pi R^2} \, d\theta \]

\[ E_x = -\frac{2k \, Q}{\pi R^2} \int_0^{\pi/2} \sin \theta \, d\theta = -\frac{2k \, Q}{\pi R^2} \]

\[ E_y = -\frac{2k \, Q}{\pi R^2} \int_0^{\pi/2} \cos \theta \, d\theta = -\frac{2k \, Q}{\pi R^2} \]

\[ E = \sqrt{E_x^2 + E_y^2} = \frac{2\sqrt{2} \, k \, Q}{\pi R^2} \]

\[ \tan \phi = \frac{E_y}{E_x} = 1 \]

\[ \phi = 45^\circ, \ 225^\circ \]
ELECTRIC FIELD LINES

(Lines of force)

a) Start on + charges or at $\infty$.

b) End on - charges or at $\infty$.

c) $\vec{E}$ at a point is a vector tangent to the electric field line at the point. It's direction is the same as that of the field line.

d) At every point the density of field lines crossing a surface at right angles to the direction of the field is proportional to the magnitude of the field.

e) The number of field lines that start or end on a point charge is proportional to the charge.
Example:

Electric Field Lines for a Point Charge
ELECTRIC DIPOLE

\[ \vec{p} = q \vec{d} \]

\[ \tau = qE \left( \frac{d}{2} \sin \phi \right) + qE \left( \frac{d}{2} \sin \phi \right) = q d E \sin \phi = p E \sin \phi \]

\[ \vec{\tau} = \vec{p} \times \vec{E} \]

\[ \tau = p E \sin \phi \]
Energy of a dipole in an electric field

Work done by an external agent to rotate a dipole from $\phi_1$ to $\phi_2$ is
\[
W = \int_{\phi_1}^{\phi_2} \tau d\phi = pE \int_{\phi_1}^{\phi_2} \sin \phi d\phi = -pE [\cos \phi_2 - \cos \phi_1]
\]

The potential energy of the dipole is
\[
U = W
\]

We choose the zero of the potential energy to be when $\vec{p} \perp \vec{E}$ ($\phi_1 = 90^\circ$). Then the potential energy at any other angle $\phi_2 \equiv \phi$ is
\[
U = -pE \cos \phi = -\vec{p} \cdot \vec{E}
\]