Single Slit Diffraction

destructive interference if

\[ \delta \equiv \frac{a}{2} \sin \theta = \frac{\lambda}{2} \]

or

\[ a \sin \theta = \lambda \]
In general, for single slit diffraction minima

\[ a \sin \theta_m = m\lambda \quad m=\pm 1, \pm 2, \pm 3, \ldots \]

Note: \( m \neq 0 \)

Note: The formulae for double slit interference and single slit diffraction are very similar

Double slit interference maxima:

\[ d \sin \theta_m = m\lambda \quad m=0, \pm 1, \pm 2, \pm 3, \ldots \]

Single slit diffraction minima:

\[ a \sin \theta_m = m\lambda \quad m=\pm 1, \pm 2, \pm 3, \ldots \]
Construction for $m=3$ case

destructive interference for

$$\delta = \frac{a}{6} \sin \theta = \frac{\lambda}{2} \quad \Rightarrow$$

$$a \sin \theta_3 = 3\lambda$$
If the slit is uniformly illuminated, the fraction of intensity in each segment $\Delta y$ is proportional to

$$\frac{\Delta y}{a}$$
The path difference for a ray at position \( y \) relative to the ray at the origin (\( y=0 \)) is

\[
\delta = y \sin \theta
\]

which corresponds to a phase shift

\[
\beta = k \delta = \frac{2\pi}{\lambda} y \sin \theta
\]

Thus, each segment \( \Delta y \) produces a field at the distant screen proportional to

\[
\Delta y \ e^{-i\beta} = \Delta y \ e^{-i \frac{2\pi}{\lambda} y \sin \theta}
\]

The total field is just proportional to the sum of these

\[
\int_{-\frac{a}{2}}^{+\frac{a}{2}} e^{-i \frac{2\pi}{\lambda} y \sin \theta} \, dy = \int_{-\frac{a}{2}}^{+\frac{a}{2}} e^{-i K y} \, dy = \frac{e^{i K \frac{a}{2}} - e^{-i K \frac{a}{2}}}{i K}
\]

\[
= \frac{2i \sin K \frac{a}{2}}{i K} = a \sin \left[ \frac{\pi a \sin \theta}{\lambda} \right]
\]

\[
= a \frac{\sin \left[ \frac{\pi a \sin \theta}{\lambda} \right]}{\lambda}
\]
The intensity is proportional to the square of the total field so we have

\[ I(\theta) = I_0 \left[ \frac{\sin \left( \frac{\pi a \sin \theta}{\lambda} \right)}{\pi a \sin \theta} \right]^2 \]
FOR DIFFRACTION:

Most of the energy lies in the central bright band.

The first order minima are at

$$\sin \theta_1 = \frac{\lambda}{a}$$

If $$\lambda << a$$,
$$\theta_1 \approx 0^\circ$$ and light travels in straight lines (sharp shadows)

If $$\lambda = a$$,
$$\theta_1 \approx 90^\circ$$ and light spreads out over the whole screen.
Diffraction Grating

Constructive interference

\[ \delta = d \sin \theta_m = m\lambda \]

\[ m = 0, \pm 1, \pm 2, \pm 3, \ldots \]