GEOMETRICAL OPTICS

Apertures and detectors large compared to $\lambda$.

Ray:
- Direction of energy flow.
- Direction of wave velocity.

Wavefront:
- Continuous set of points of equal amplitude.
- Continuous surface that is in the same phase of motion.

Must always have ray $\perp$ wavefront
To see an object:

Light must leave the object and enter your eye.

Light leaves objects by:

a) radiation - e.g. sun, light bulb, ...

b) reflection
Law of Reflection:

angle of incidence = angle of reflection

$\theta_i = \theta_r$
Specular Reflection - smooth surface

Diffuse Reflection - rough surface

One sees most objects via diffuse reflection
Spherical Mirror

A curved surface that is a section out of a sphere.

The dash-dot line is the principle axis - (the axis of symmetry)

C is the center of curvature of the mirror. The distance between C and any point on the mirror surface is R.

F is the focal point - A ray parallel to the principle axis is reflected through F.
A ray passing through C will be reflected back on itself.
A ray parallel to the principle axis will be reflected through F.

\[
\sin \varphi = \frac{h}{R} \quad \cos \varphi = \sqrt{1 - \frac{h^2}{R^2}}
\]

\[
\cos \varphi = \frac{R/2}{R-f}
\]

\[
\frac{R/2}{R-f} = \sqrt{1 - \frac{h^2}{R^2}}
\]
\[ R - f = \frac{R}{2} \frac{1}{\sqrt{1 - \frac{h^2}{R^2}}} \]

\[ = \frac{R}{2} \left( 1 + \frac{h^2}{2R^2} + \ldots \right) \]

\[ f \approx R - \frac{R}{2} \left( 1 + \frac{h^2}{2R^2} \right) \]

\[ f \approx \frac{R}{2} - \frac{1}{4} \frac{h^2}{R} \]

Finally, for \( h \ll R \)

\[
\begin{array}{c}
    f \approx \frac{R}{2}
\end{array}
\]
Image construction by

RAY TRACING

Draw two rays from a point on the object, the point where they intersect is the image point.

RAY 1 : From the point on the object through the center of curvature C. The ray is reflected back on itself.

RAY 2 : From the point on the object, parallel to the principle axis. The ray is reflected through the focal point.
Thus,

\[ \frac{-h_i}{h_o} = \frac{d_i}{d_o} \]

\[ \frac{-h_i}{h_o} = \frac{R - d_i}{d_o - R} \]

Thus,

\[ \frac{d_i}{d_o} = \frac{R - d_i}{d_o - R} \]
\[ d_i d_o - R d_i = R d_o - d_i d_o \]

\[ R d_o + R d_i = 2 d_i d_o \]

Multiply by \( \frac{1}{R d_i d_o} \)

\[ \frac{1}{d_o} + \frac{1}{d_i} = \frac{2}{R} \]

Since \( f = \frac{R}{2} \) and we have the Mirror Equation

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]

Magnification is

\[ m = \frac{\text{image height}}{\text{object height}} = \frac{h_i}{h_o} \]

\[ m = -\frac{d_i}{d_o} \]
Sign Convention

(For Mirrors, Lenses, and Refracting Surfaces)

Note:

a) There is a side for which the light is incoming.

b) There is a side for which the light is outgoing.

For a mirror these are the same side!

1. Object on incoming side:
   \[ d_o > 0 \text{ (real object)}; \text{ otherwise} \]
   \[ d_o < 0 \text{ (virtual object)} \]

2. Image on outgoing side:
   \[ d_i > 0 \text{ (real image)}; \text{ otherwise} \]
   \[ d_i < 0 \text{ (virtual image)} \]

3. Center of curvature on outgoing side:
   \[ R > 0 \text{ (concave mirror)}; \text{ otherwise} \]
   \[ R < 0 \text{ (convex mirror)} \]
Sign convention via a picture.

d_o, d_i, R, and f are all positive in this picture. If any one of these is on the opposite side of the mirror, it will be negative.
Plane Mirror

\[ R = \infty \quad f = \infty \]

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{\infty} = 0
\]

\[ d_o = -d_i \]

\[ m = -\frac{d_i}{d_o} = +1 \]
EXAMPLE:

20 inch diameter silver ball

\[ d_o = 10'' \quad r = -10'' \quad f = -5'' \]

\[
\frac{1}{10} + \frac{1}{d_i} = \frac{1}{-5} \\
\frac{1}{d_i} = \frac{-1}{5} - \frac{1}{10} = \frac{-2}{10} - \frac{1}{10}
\]

\[ d_i = -\frac{10}{3} = 3.33'' \]

\[ m = -\frac{d_i}{d_o} = -\frac{-10/3}{10} = +\frac{1}{3} \]