The Magnetic Field
Fig. 27.1 "Likes" Repel and "Unlikes" Attract

- Repulsive
- Attractive
Fig. 27.5 Magnetic Field of Earth

North geographic pole
Magnetic pole
Compass
Magnetic pole
South geographic pole
left-hand rule

right-hand rule

\[ \vec{F} = q\vec{v} \times \vec{B} \]
Right-hand rule

\[ \vec{F} = q\vec{v} \times \vec{B} \]
The force \( \vec{F} \) on a charge \( q \) moving with a velocity \( \vec{v} \)

\[
\vec{F} = q\vec{v} \times \vec{B}
\]

vector product rules

\[
x, y, z \quad y, z, x \quad z, x, y \quad i, j, k
\]

\[
x \times y = z \quad y \times z = x \quad z \times x = y
\]

\[
y \times x = -z \quad z \times y = -x \quad x \times z = -y
\]

\[
x \times x = 0, \quad etc
\]

right-hand rule
The force $\vec{F}$ on a charge $q$ moving with a velocity $\vec{v}$ is given by

$$\vec{F} = q\vec{v} \times \vec{B}$$

The magnitude of the force: $|F| = qvB \sin \theta$

- **zero magnitude force**
  
  \[ \vec{v} \parallel \vec{B} \quad F = 0 \]

- **maximal magnitude of the force**
  
  \[ F_{\text{max}} = qvB \]

- **intermediate magnitude**
  
  \[ F = qvB \sin \theta \]
Units of the magnetic fields

The force \( \vec{F} \) on a charge \( q \) moving with a velocity \( \vec{V} \)

The magnitude of the force \( F = qvB \sin \theta \)

\[ [B] = \text{Newtons} / (\text{Coulomb} \cdot \text{meter} / \text{sec}) \]

\[ 1T(\text{tesla}) = 1N / C \cdot m / s = 1N / A \cdot m \]

\[ 1T(\text{tesla}) = 10^4 G(\text{gauss}) \]

\[ B_{\text{Earth}} \approx 1 \text{Gauss} = 10^{-4} T \]
The charge is negative; force acts down – path 3

\[ \vec{F} = q\vec{v} \times \vec{B} \]
The force is always perpendicular to velocity,

The work done by the magnetic force is zero!

Therefore magnetic field cannot change the magnitude of the velocity, only its direction.

Motion of a charged particle under the action of a magnetic field alone is always motion with a constant \(|\text{absolute value of velocity}|\).
Uniform magnetic field, $\vec{v} \perp \vec{B}$
When a charged particle has velocity components both perpendicular and parallel to a uniform magnetic field, the particle moves in a helical path. The magnetic field does no work on the particle, so its speed and kinetic energy remain constant.
\[ \vec{F} = q\vec{v} \times \vec{B} \]

\[ \vec{F} = m\vec{a} \quad F_r = ma_r \quad F_\theta = 0 \]

\[ \vec{v} \perp \vec{B} \quad F = qvB \]

\[ -qvB = -mr\omega^2 = -m \frac{v^2}{r} \]

\[ r = \frac{mv}{qB} \]

\[ \omega = \frac{v}{r} = \frac{v}{\left( \frac{mv}{qB} \right)} = \frac{qB}{m} \]
Angular velocity of rotation of a charge in a magnetic field
(cyclotron frequency)

\[ f = \frac{\omega}{2\pi} \]

\[ T = \frac{2\pi r}{v} \quad f = \frac{1}{T} \text{Hz (hertz)} \quad \omega = \frac{2\pi}{T} \text{(radian per sec)} \]

\[ \omega = \frac{v}{r} = \frac{v}{\left( \frac{mv}{qB} \right)} = \frac{qB}{m} \]

\[ r = \frac{mv}{qB} \]

\[ f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m} \]
Example 27.3

Electron motion in a microwave oven

A magnetron in a microwave oven emits electromagnetic waves with frequency $f=2450\ MHz$. What magnetic field strength is required for electrons to move in circular paths with this frequency?

$$\omega = 2\pi f = 2\pi (2450 \times 10^6) s^{-1} = 1.54 \times 10^{10} s^{-1}$$

$$B = \frac{m\omega}{q} = \frac{9.11 \times 10^{-31} kg}{1.6 \times 10^{-19} C} \times 1.54 \times 10^{10} s^{-1} = 0.088 T$$

Incredibly:
electrons revolve in the tiny field of the Earth with few millions rotations per second.
Trapped particles: a magnetic bottle.

Particle near the end experiences a force toward the center.

A pendulum in which oscillations are between the radial and longitudinal (horizontal) velocities. In the center the longitudinal velocity is larger, while rotation is slower. At the edge the particle rotates faster. **The total kinetic energy is conserved!**
Trapped particles: Van Allen’s radiation belts. Why radiation? What are: synchrotron, microwave oven, radiation belts?

In fact, radiation belts are “anti-radiation”: they protect us against cosmic radiation coming mostly from the Sun.
Magnetic field traps charged particles
Northern lights

Aurora borealis
Atoms are made with positive protons in the nucleus and with electrons outside.

What is bad in the picture of electrons rotating around the atomic nuclear, like planets around the Sun?

The electrons will radiate like in the microwave oven and, therefore, will lose their energy. Such an atom will be unstable.

**Death and life question:** if rotating electrons radiate why atoms are stable? May be electrons and protons are in a static configuration?
Using Crossed $\vec{E}$ and $\vec{B}$ Fields

Velocity selector

$$qvB - qE = 0$$

$$E = vB$$

$$v = \frac{E}{B} \quad \text{independent of the mass of the particle!}$$
Between plates $P$ and $P'$ there are mutually perpendicular, uniform $\vec{E}$ and $\vec{B}$ fields.
Thomson’s e/m experiment

1897: Cavendish Laboratory in Cambridge, England

\[ \frac{1}{2} mv^2 = eV \Rightarrow v = \sqrt{\frac{2eV}{m}} \]

\[ v = \frac{E}{B} \]

\[ \frac{E}{B} = \sqrt{\frac{2eV}{m}} \Rightarrow e = \frac{E^2}{2VB^2} \]
Mass spectrometer (1919)

\[ R_1 = \frac{m_1 v}{qB} \]

\[ v = \frac{E}{B} \quad R = \frac{mv}{qB} \]

\[ R_1 = \frac{m_1 E}{qB^2} \]

\[ R_2 = \frac{m_2 E}{qB^2} \]
Magnetic Force on a Current-Carrying Conductor

\[ F = (nAl)(qv_d B_\perp) = (nqv_d)AlB_\perp = jAlB_\perp = IlB_\perp \]

Decisive moment: vector's direction was delegated from \( v \) to \( dl \)
Magnetic Force on a Current-Carrying Conductor

Decisive moment: vector’s direction was delegated from $\vec{v}$ to $dl$

\[
d\vec{F} = (nAdl)\left(q\vec{v}_d \times \vec{B}\right) = (qn\vec{v}_d A)dl\left(\vec{u}_d \times \vec{B}\right) = I\vec{d}l \times \vec{B}
\]

here $\vec{u}_d = \vec{v}_d / v_d$ is direction of motion along $dl$;

then, the direction was delegated from $\vec{v}_d$ to $d\vec{l}$
Magnetic Force on a Curved Conductor
Example 27.8

\((-x) \times z = y\)

\[ \mathbf{F} = I(L + 2R)B\hat{y} \]

\[ d\mathbf{F} = I dl \times \mathbf{B} \]

\[ dF = I(R d\theta)B \]

\[ dF_x = IR(d\theta)B \cos \theta \]

\[ dF_y = IR(d\theta)B \sin \theta \]

\[ F_x = IRB \int_0^\pi \cos \theta \, d\theta = 0 \]

\[ F_y = IRB \int_0^\pi \sin \theta \, d\theta = 2IRB \]
Check that you understand why the resulting force depends ONLY on the position of the two ending points, but not on the shape of the curved conductor!

\[ d\vec{F} = I dl \times \vec{B} \]
\[ \vec{F} = I \int_{\text{along current}} d\vec{l} \times \vec{B} = \]
\[ = I \left( \int_{\text{along current}} d\vec{l} \right) \times \vec{B} = \]
\[ = I \vec{L}_I \times \vec{B} \]
\[ \vec{L}_I = r_{\text{final}} - r_{\text{initial}} \]

Counted according to the direction of current

\[ \vec{F} = I(L + 2R)\hat{y} \]
Rail-gun (a primitive motor)

\[
\text{power (mechanical)} = \vec{F} \cdot \vec{v}
\]

To understand how does a balance between mechanical and electric powers work - see Chapter 29
Torque acting on a loop carrying current

\[ \tau = 2IBa\left(\frac{b}{2}\sin \phi\right) \]

\[ = IBab \sin \phi \]

\[ \vec{\mu} = I(d\vec{S}) \]
Magnetic (and electric) Torque and Potential Energy $U$ for a Dipole

\[ \tau = IBab \sin \phi = B(Iab) \sin \phi = \mu B \sin \phi \]

- **magnetic** \( \vec{\tau} = \vec{\mu} \times \vec{B} \)
- **electric** \( \vec{\tau} = \vec{p} \times \vec{E} \)

**potential energy (m)** \( U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \phi \)

**potential energy (el)** \( U = -\vec{p} \cdot \vec{E} = -pE \cos \phi \)
Attraction of a Solenoid by a Magnet; 
final orientation is such that the torque becomes minimal

Solenoid is a magnet itself. Its direction from South to North is along $\mu$. Naturally, the North of the solenoid attracts to the South of the magnet.

Potential energy (magnetic):

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \phi$$
Solenoid as a Magnet; the torque tries to orient the solenoid along the magnetic field

Example 27.9

\[
\vec{\mu} = I(d\vec{S})
\]

magnetic moment \( \mu = N_{\text{coils}} \pi r^2 I \)

\[
\tau = \mu B \sin \phi
\]

Solenoid is a magnet itself. Its direction from South to North is along \( \vec{\mu} \). Naturally, the North of the solenoid attracts to the South of the magnet.
How a dc motor works

(a) Brushes are aligned with commutator segments.
- Current flows into the red side of the rotor and out of the blue side.
- Therefore the magnetic torque causes the rotor to spin counterclockwise.

(b) Rotor has turned 90°.
- Each brush is in contact with both commutator segments, so the current bypasses the rotor altogether.
- No magnetic torque acts on the rotor.

(c) Rotor has turned 180°.
- The brushes are again aligned with commutator segments. This time the current flows into the blue side of the rotor and out of the red side.
- Therefore the magnetic torque again causes the rotor to spin counterclockwise.

In what position the torque is maximal? minimal?
A series DC motor: What happens when a motor suddenly stops? Did it stop because it was burnt, or it was burnt because it had been stopped by a jam?

To understand what is going on here see Example 29.5. It helps to understand what happens when the motor stops.
Additional material 1
Gauss’s law for Magnetism

\[ \oint_S \vec{B} \cdot d\vec{S} = 0 \]

magnetic charges do not exist

magnetic lines have no ends

Some magnetic field lines remind electric dipole. Where is the difference?.
The magnetic field lines

27.14  (a) Like little compass needles, iron filings line up tangent to magnetic field lines.  
(b) Drawing of the field lines for the situation shown in (a).

Magnetic field pattern reminds 
that of an electric dipole.  
It is illusive! Why?
Trapped particles: a magnetic bottle.

Particle near the end experiences a force toward the center.

Controlled thermo-nuclear reaction.

Question: in what direction revolves a trapped charge.

bounce motion
Additional material 2
Hall effect:

In a magnetic field, there arises an electric field inside a current-carrying conductor that is perpendicular to the directions of the current and the magnetic field. The sign of the induced electric field (which in its turn induces a potential difference, i.e., the Hall voltage) depends on whether the mobile charges are positive or negative.

The effect can be used to determine the sign of the charge-carries.
\[ E_{\text{Hall}} = \frac{IB}{nAq} = \frac{jB}{nq} \]