• There is no electric field inside a conductor
• Net charge can only reside on the surface of a conductor
• Any external electric field lines are perpendicular to the surface (there is no component of electric field that is tangent to the surface).

• The electric potential within a conductor is constant

Caution:
all above is valid only in the absence of electric currents!
Definition of the electric current

\[ I = \frac{\Delta Q}{\Delta t} \]

\[ I(t) = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt} \]
Direction of the electric current

current density: \( \vec{j} = qn_\text{el} \vec{v} \)

current density of electrons: \( en_\text{el} \vec{v} \)
Symbols for the circuit diagrams

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<td>Ammeter (measures current through it)</td>
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Symbols for the circuit diagrams

\[ V_{ab} = V \]

Voltmeter has very large resistance  
(should be switched on in parallel)

Ammeter has very small resistance  
(should be switched on in series)

in both cases the purpose is to **minimize** the influence of the device on the circuit
Current density; current as a flux through a cross-section

Current as a flux: consider current flowing in an inhomogeneous wire

\[ I = \int \vec{j} \cdot d\vec{S} \]

Current flowing in a homogeneous wire with a cross-sectional area \( A \).

\[ I = \int_A \vec{j} \cdot d\vec{S} = \int_A j dS = jA \]

\[ j = I / A \]
Ohm’s Law

Ohm’s Law: \[ I = \frac{V}{R} \]; \( R = \text{const} \) (i.e. independent of \( V \))

Local form of the Ohm’s law.

\[ \frac{1}{\rho} \vec{E} = \vec{j} \]  \( \rho \) is resistivity

in a homogeneous medium:

\[ \frac{1}{\rho} \vec{E} = \vec{j} \Rightarrow \frac{1}{\rho \ L} = \frac{I}{A} \Rightarrow I = \frac{A}{\rho L} V = \frac{V}{R} \]

\[ R = \rho \frac{\text{Length}}{\text{cross-section's area}} = \rho \frac{L}{A} \]
Units

\[ I = \frac{\Delta Q}{\Delta t} \quad \text{1 Ampere} = \frac{1 \text{ Coulomb}}{1 \text{ sec}} = 1 \text{ C/s} \]

Ohm's Law: \[ \frac{V}{I} = R \]

units: \[ [R] = \frac{\text{Volt}}{\text{Ampere}} = \text{Ohm} \]

\[ \frac{1 \text{ V}}{1 \text{ A}} = 1 \Omega \]
Local forms of the Ohm’s law:

\[ \frac{1}{\rho} \vec{E} = \vec{j} \quad \rho \text{ is resistivity} \]

\[ R = \rho \frac{\text{Length}}{\text{cross-section's area}} = \rho \frac{L}{A} = \frac{L}{\sigma A} \]

Units:

\[ [\rho] = \Omega m \quad [\sigma] = (\Omega m)^{-1} \]
Ohm’s Law

Local form of the Ohm’s law. \( \frac{1}{\rho} \vec{E} = \vec{j} \) \( \rho \) is resistivity

Resistivity dependence only on the properties of the material

Resistance, unlike resistivity, depends on the geometry (including the shape and dimensions)

\[ R = \rho \frac{\text{length}}{\text{cross-section's area}} = \rho \frac{L}{A} \]

similarity between \( R \) and \( 1/C \)

\[ C^{-1} = \frac{d}{\varepsilon_0 A} \]
Why it is obvious: capacitors in series and in parallel

\[ C = \frac{Q}{\Delta V} = \varepsilon_0 \frac{A}{d} \]

\[ C = \frac{Q}{\Delta V} = \varepsilon_0 \frac{A}{d} \implies \varepsilon_0 \frac{A_1 + A_2 + \ldots}{d} \]

Capacitors in parallel: \[ C_{tot} = C_1 + C_2 + C_3 + \ldots \]

\[ C = \frac{Q}{\Delta V} = \varepsilon_0 \frac{A}{d} \implies \varepsilon_0 \frac{A}{d_1 + d_2 + \ldots} \]

Capacitors in series: \[ \frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \]
It is obvious: resistances in series and in parallel

\[ R = \frac{\Delta V}{I} = \rho \frac{L}{A} \quad \Rightarrow \quad \rho \frac{L_1 + L_2 + \ldots}{A} \]

Resistances in series: \( R_{\text{tot}} = R_1 + R_2 + R_3 + \ldots \)

\[ R = \frac{\Delta V}{I} = \rho \frac{L}{A} \quad \Rightarrow \quad \rho \frac{L}{A_1 + A_2 + \ldots} \]

Resistances in parallel:

\[ \frac{1}{R_{\text{tot}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \]
Similarity between resistance and capacitance

\[ R = \rho \frac{L}{A} \iff \frac{1}{C} = \frac{d}{\varepsilon A} \]

\[ R \iff 1/C \quad \rho \iff 1/\varepsilon \]

Similarity but not equivalence!!!
Please, don’t put \( \varepsilon \) in the formula for resistance!!!
Why it is obvious: capacitances/resistances in series and in parallel

\[ \Delta V = RI \quad \Leftrightarrow \quad \Delta V = \frac{Q}{C} \]

consider \( I \) like \( Q \) \quad \( R \Leftrightarrow 1/ C \)

\[ R = \rho \frac{L}{A} \quad \Leftrightarrow \quad \frac{1}{C} = \frac{d}{\varepsilon A} \]

Resistances (capacitances) in series:
current (charge) is fixed, voltage drops are added

Resistances (capacitances) in parallel:
voltage is fixed, currents (charges) are added
Why it is opposite: capacitances/resistances in series and in parallel

\[ \Delta V = RI \iff \Delta V = \frac{Q}{C} \]

\[ R \iff 1/C \]

It is just the difference in definitions of the Resistances and Capacitances nothing more !!!
Very misleading example:
radial flow of current

$A = 2\pi rL$

Here $L$ stands not as a length but is related to a width of a crosssection.

\[
R = \rho \int_{a}^{b} \frac{dr}{2\pi rL} = \frac{\rho}{2\pi L} \ln \frac{b}{a}
\]

Instead of a length, integration over $dr$. 
Comparison of coaxial cable (capacitor) with a radial flow of current

A long cylindrical capacitor. The linear charge density \( \lambda \) is assumed to be positive in this figure. The magnitude of charge in a length \( L \) of either cylinder is \( \lambda L \).

\[
V(\vec{r}) = \int_{r_a}^{r_b} \vec{E} \cdot d\vec{r} = \int_{r_a}^{r_b} \frac{\lambda}{2\pi \varepsilon_0} \frac{dy}{y} = \frac{\lambda}{2\pi \varepsilon_0} \ln \frac{y}{r_a} \bigg|_{r_a}^{r_b} = \frac{\lambda}{2\pi \varepsilon_0} \ln \frac{r_b}{r_a}
\]

The capacitance per unit length is

\[
C = \frac{Q}{V_{ab}} = \frac{\lambda L}{2\pi \varepsilon_0 \ln \frac{r_b}{r_a}} = \frac{2\pi \varepsilon_0 L}{\ln \left( \frac{r_b}{r_a} \right)}
\]

\[
R = \rho \int_a^b \frac{dr}{2\pi r L} = \frac{\rho}{2\pi} \ln \frac{b}{a}
\]
Equivalent resistance
Kirchhoff's rule: $12V - 2\Omega \times 2A - 4\Omega \times 2A = 0$
Power output

\[ P = \frac{dW}{dt} \quad W = QV \quad \Rightarrow \]

\[ P = V dQ / dt = IV \]

\[ P = IV = R I^2 = \frac{V^2}{R} \]

\( (V = \Delta V_{ab}) \)

The current in a 100-W light bulb is

\[ I = \frac{P}{V} = \frac{100\, W}{120\, V} = 0.83\, A \]

\[ R = \frac{V}{I} = \frac{120\, V}{0.83\, A} = 144\, \Omega \]
Power input and output in a complete circuit

\[ I = 2 \, A \quad E_{\text{batt}} I = 12V \cdot 2A = 24W \]

\[ P_r = I^2r = (2A)^2 (2\Omega) = 8W \]

\[ P_{ab} = V_{ab} I = 8V \cdot 2A = 16W \]

Power in a short circuit

\[ I = 6 \, A \quad E_{\text{batt}} I = 12V \cdot 6A = 72W \]

\[ P_r = I^2r = (6A)^2 (2\Omega) = 72W \]
Power output from a battery to a headlight

\[ P = \frac{dW}{dt} \quad W = V_{ab} \int Q \quad \Rightarrow \]

\[ P = V_{ab} \cdot \frac{dQ}{dt} = V_{ab} \cdot I \]

\[ E_{batt} = I r_{int} + V_{ab} \]

\[ P_{h\text{-light}} = IV_{ab} \]

\[ P_{h\text{-light}} = IV_{ab} = E_{batt} I - r_{int} I^2 \]

\[ P_{batt} = P_{h\text{-light}} + P_{\text{internal heating}} \]
Power output from a battery to a headlight (details)

\[ V_{ab} = E_{batt} - I r_{\text{int}} \]

\[ P_{h\text{-light}} = IV_{ab} \]

\[ P_{h\text{-light}} = IV_{ab} = E_{batt} I - r_{\text{int}} I^2 \]

\[ I = \frac{E_{batt}}{R + r_{\text{int}}} \]

\[ P_{h\text{-light}} = E_{batt}^2 \frac{R}{(R + r_{\text{int}})^2} \]

\[ P_{\text{internal-heating}} = E_{batt}^2 \frac{r_{\text{int}}}{(R + r_{\text{int}})^2} \]

\[ P_{\text{batt}} = P_{h\text{-light}} + P_{\text{internal-heating}} \]

\[ = E_{s}^2 \frac{1}{R + r_{\text{int}}} \]
Power input to a battery from a source (an alternator);

pay attention that the current **now** comes **into** the battery from plus to minus

\[
E_{\text{batt}} - I r_{\text{int}} - V_{ab} = 0
\]

\[
V_{ab} = E_{\text{batt}} + I r_{\text{int}}
\]

\[
P_{\text{source}} = IV_{ab}
\]

\[
P_{\text{source}} = IV_{ab} = EI + r_{\text{int}} I^2
\]

\[
P_{\text{source}} = P_{\text{charging}} + P_{\text{heating}}
\]
Power input to a battery (continuation)

\[ V_{ab} = E_{batt} + I r_{\text{int}} \]

\[ P_{\text{source}} = IV_{ab} \]

\[ P_{\text{source}} = IV_{ab} = E_{batt} I + r_{\text{int}} I^2 \]

\[ I = \frac{V_{ab} - E_{batt}}{r_{\text{int}}} \]

\[ P_{\text{charging}} = E_{batt} I = E_{batt} \frac{V_{ab} - E_{batt}}{r_{\text{int}}} \]

\[ P_{\text{source}} = P_{\text{charging}} + P_{\text{batt-heating}} \]
Voltmeter has very large resistance (should be switched on in parallel)

Ammeter has very small resistance (should be switched on in series)

in both cases the purpose is to *minimize* the influence of the device on the circuit
Kirchhoff’s rules

26.7 (a) Kirchhoff’s junction rule states that as much current flows into a junction as flows out of it. (b) A water-pipe analogy.

(a) Kirchhoff’s junction rule

Junction

\[ I_1 \rightarrow I_2 \]

\[ I_1 + I_2 \]

(b) Water-pipe analogy for Kirchhoff’s junction rule

The flow rate of water leaving the pipe equals the flow rate entering it.

Kirchhoff's rules:

\[ \sum_{\text{loop}} V = 0 \]

\[ \sum_{\text{in}} I = \sum_{\text{out}} I \]
Kirchhoff’s rules

25.24 When two sources are connected in a simple loop circuit, the source with the larger emf delivers energy to the other source.

26.8 Use these sign conventions when you apply Kirchhoff’s loop rule. In each part of the figure “Travel” is the direction that we imagine going around the loop, which is not necessarily the direction of the current.

Kirchhoff’s rule:

\[ V_{ab} - E_{batt} - Ir_{int} = 0 \]
Example 26.3

Caution (wrong illustration): starting an engine is not charging a dead battery! but polarities are shown correctly

\[ K - \text{ff}'s \ rule : 12V - 2\Omega \times I - 3\Omega \times I - 4\Omega \times I - 4V - 7\Omega \times I = 0 \]
Example 26.4

Caution:
In this example the actual polarity of the battery is opposite to that shown in Fig. 26.11

\[ K - f f 's \text{ rules:} \]

(1) \[ 12V - r_{\text{int}} \times 3A - 3\Omega \times 2A = 0 \quad r_{\text{int}} = 2\Omega \]

(2) \[ -3\Omega \times 2A - E_{\text{batt}} + 1\Omega \times 1A = 0 \quad E_{\text{batt}} = -5V \]

(3) check: \[ 12V - 2\Omega \times 3A - 1\Omega \times 1A + (-5V) = 0 \]
Kirchhoff’s rules, Example 26.6

\[ K - \phi 's \text{ rules, loop 1: } 13V - 1\Omega \times I_1 - 1\Omega \times (I_1 - I_3) = 0 \]

\[ \text{loop 2: } 13V - 1\Omega \times I_2 - 2\Omega \times (I_2 + I_3) = 0 \]

\[ \text{loop 3: } -1\Omega \times I_1 - 1\Omega \times I_3 + 1\Omega \times I_2 = 0 \]

\[ I_1 = 6A; \ I_3 = -1A; \ I_2 = 5A \]

check loop 4: \[-1\Omega \times (-1A) - 2\Omega \times 4A + 1\Omega \times 7A = 0 \]
Kirchhoff’s rules, Example 26.6

Look on this scheme as going down from the 13th floor (point c; 13volt) to the ground floor (point d; 0volt).

On what floor is point a? point b? What floor is higher? In what direction flows current between points a and b?
Discharging a capacitor

$I(t) = \frac{dq}{dt} < 0$

$RI(t) + \frac{q(t)}{C} = 0$

basic equation: $R \frac{dq}{dt} + \frac{q(t)}{C} = 0$

solution:

$q(t) = Q_{\text{initial}} \exp\left[-\frac{(t - t_{\text{initial}})}{RC}\right]$

caution: be careful with the direction/sign of current
Charging a capacitor

they assume that initial charge is zero:

\[ Q_{\text{initial}} = 0 \]

does this maybe not so
Charging a capacitor

Kirchhoff's rule:

\[ E_{batt} - IR - q / C = 0 \]

\[ I = \frac{dq}{dt} = \frac{E_{bat}}{R} - \frac{q}{RC} \quad \Rightarrow \quad \frac{dq}{dt} = -\frac{1}{RC} (q - CE_{bat}) \]

\[ q(t) - CE_{bat} = \tilde{q}(t) \quad q(t) = \tilde{q}(t) + CE_{bat} \]

\[ \tilde{q}(t \to \infty) \to 0 \] correspondingly, \( q(t \to \infty) = CE_{bat} = Q_{final} \)

\[ Q(t = t_{initial}) = Q_{initial} \]

\[ q(t) = Q_{final} + (Q_{initial} - Q_{final}) \exp[-(t - t_{initial}) / RC]\]

\[ I(t) = \frac{Q_{final} - Q_{initial}}{RC} \exp[-(t - t_{initial}) / RC] \]
Kirchhoff’s rules in charging capacitor

\[ E = E_{\text{batt}} \]

**K - ff's rules, loop 1:** \[ E - I_1 r - \frac{Q}{C} = 0 \quad \Rightarrow \quad I_1 = \frac{1}{r} \left( E - \frac{Q}{C} \right) \]

**loop 2:** \[ E - I_1 r - I_2 R = 0 \quad \Rightarrow \quad I_2 = \frac{Q}{CR} \]

**step 3:** \[ \frac{dQ}{dt} = I_1 - I_2 \]

**equation:** \[ \frac{dQ}{dt} = \frac{1}{r} \left( E - \frac{Q}{C} \right) - \frac{Q}{CR} \quad \Rightarrow \quad Q(t) = Q_{\text{final}} \left( 1 - \exp(-t / R\|C) \right) \]

\[ Q_{\text{final}} = EC \frac{R}{R + r}; \quad R\| = \frac{Rr}{R + r} \]

“From the point of view of the capacitor”, resistances are in parallel
Q 25-13
Why does a light bulb burn out when you switch the light on and never when you turn it off?
Resistivity and temperature

metal:
\[ \rho \] increases with increasing \( T \)

semiconductor:
\[ \rho \] decreases with increasing \( T \)
Q 25-13
Why does a light bulb burn out when you switch the light on and never when you turn it off?

Answer:
when lamp’s wire is cold its resistance is smaller; what happens is similar to a short-circuit with a weak spot acting like a fuse!
Wheatstone bridge

\[ V_D = V_B \implies I_g = 0 \]
\[ R_x = \frac{R_2 R_3}{R_1} \]