PHYSICS 218 FINAL EXAM
Spring, 2005

Name: Solutions
Signature: _______________________
Student ID: ____________________
E-mail: _______________________
Section Number: _____________

• You have the full class period to complete the exam.
• Formulae are provided on the last page. You may NOT use any other formula sheet.
• When calculating numerical values, be sure to keep track of units.
• You may use this exam or come up front for scratch paper.
• Be sure to put a box around your final answers and clearly indicate your work to your grader.
• All work must be shown to get credit for the answer marked. If the answer marked does not obviously follow from the shown work, even if the answer is correct, you will not get credit for the answer.
• Clearly erase any unwanted marks. No credit will be given if we can’t figure out which answer you are choosing, or which answer you want us to consider.
• Partial credit can be given only if your work is clearly explained and labeled.

Put your initials here after reading the above instructions:
Part I. Conceptual Questions (5 pts each)
Circle the correct answer

1. A ball is thrown up in the air. At the highest point the ball reaches,
   (a) its acceleration is zero
   (b) its acceleration is horizontal
   (c) its acceleration is vertically down
   (d) its acceleration is vertically up

2. If an object is in simple harmonic motion, its:
   (a) Velocity is proportional to its displacement.
   (b) Acceleration is proportional to its displacement.
   (c) Kinetic energy is proportional to its displacement.
   (d) Period is proportional to its displacement.
   (e) Frequency is proportional to its displacement.

3. Haley’s comet has an elliptical orbit around the sun, as shown in the figure at right. At which point in its orbit does it have its greatest angular momentum with respect to the Sun’s center? (Neglect the effect of the planets).
   (a) Point A
   (b) Point B
   (c) Point C
   (d) Point D
   (e) Its angular momentum is the same everywhere.

4. In the problem above, at which point in its orbit does it have its greatest angular velocity?
   (a) Point A
   (b) Point B
   (c) Point C
   (d) Point D
   (e) Its angular velocity is the same everywhere.

5. When a car moves in a circle with constant speed, its acceleration is:
   (a) constant in magnitude and pointing towards the center
   (b) constant in magnitude and pointing away from the center
   (c) zero
   (d) constant in magnitude and direction
   (e) none of these
Continued from part I:
6. If a body moves in such a way that its linear momentum is constant, then
   (a) its kinetic energy is zero.
   (b) the net force acting on it must be zero.
   (c) its acceleration must be nonzero and constant.
   (d) its center of mass must remain at rest.
   • frictional forces must be negligible.
7. You are using a wrench and trying to loosen a rusty nut. Which of these arrangements will be the most effective in loosening the nut? (The diagrams are looking from below the nut.)
   a. b. c. d.
8. You and your friend are riding on a merry-go-round that is turning. If your friend is twice as far as you are from the merry-go-round’s center, and, if you and he are both of equal mass, which statement is true about your friend’s moment of inertia with respect to the axis of rotation?
   • It is four times yours.
   • It is twice yours.
   • It is the same for both of you.
   • Your friend has greater moment of inertia but it is impossible to say how much more than yours it is.
9. The ratio of the gravitational force at an altitude 3 \( R_E \) above the Earth’s surface (\( R_E = \) the radius of the Earth) to the gravitational force at the Earth’s surface is
   (a) \( \frac{1}{16} \)
   (b) \( \frac{1}{9} \)
   (c) 3
   (d) \( \frac{1}{4} \)
10. By what factor will the period of a pendulum change as you take it to an altitude 3 \( R_E \) above the surface of the earth?
    (a) 2
    (b) 4
    (c) \( \frac{1}{4} \)
    (d) \( \frac{1}{3} \)
Problem 2: Spacecraft in an elliptical orbit (30 points)

A spacecraft of mass \( m \) in an elliptical orbit around the Earth has a low point in its orbit (perigee) a distance \( R_p \) from the center of the Earth; at its highest point (apogee) the spacecraft is a distance \( R_a \) from the center of the Earth. In terms of \( m, R_p, R_a, G, \) and \( M_E \)

a) (3 pts) Draw the free body diagram for the spacecraft at one of these points?

b) (7 pts) Using conservation of angular momentum, find the ratio of the speeds at apogee and perigee.

c) (10 pts) Using conservation of energy, find the speed at apogee and at perigee.

d) (5 pts) What is the work necessary to escape the Earth completely if we fire rockets at apogee?

e) (5 points) What is the work necessary to escape the Earth completely if we fire rockets at perigee? Is it greater than at apogee?

\[ a) \]

\[ \text{Diagram showing a spacecraft orbiting Earth.} \]

\[ b) \]

\[ m v_p R_p = m v_a R_a \Rightarrow \frac{v_a}{v_p} = \frac{R_p}{R_a} \]

\[ c) \]

\[ \frac{1}{2} m v_p^2 - \frac{6 M_G m}{R_p} = \frac{1}{2} m v_a^2 - \frac{6 M_G m}{R_a} \]

\[ \Rightarrow \frac{1}{2} \left(1 - \frac{R_p^2}{R_a^2}\right) v_p = -6 M_E \left(\frac{1}{R_a} - \frac{1}{R_p}\right) \]

\[ v_p = \sqrt{\frac{2 G M_E \left(\frac{1}{R_a} - \frac{1}{R_p}\right)}{\left(1 - \frac{R_p^2}{R_a^2}\right)}} = \sqrt{\frac{2 G M_E R_a}{R_p (R_a + R_p)}} \quad \text{with} \quad v_a \text{ same with } - v_p \]

d\&e) \quad W = \Delta KE. \text{ The } \Delta KE \text{ needed to escape is equal to the potential energy} \]

\[ W = \frac{6 M_E m}{R_p} - \frac{1}{2} m 2 G M_E R_a = \frac{6 M_E m}{R_a + R_p} = W \]
Problem 3: Going up a plane (35 points)

A disk with uniform density and $M$ and radius $R$ is rolling without slipping with constant speed $V$ along a flat horizontal surface. It then goes up an inclined plane of angle $\theta$ as shown in the figure. Assume the acceleration due to gravity is given by $g$.

$I_{\text{disk}} = \frac{1}{2}MR^2$

a) (10 pts) How high up the inclined plane will the ball go? In other words, what is the height $h$ in the figure?

b) (10 pts) Draw the force diagram of the disk as it is going up the ramp.

c) (10 pts) Write down Newton's 2nd law equations (linear and angular) of motion as the disk is going up the ramp.

b) 

\[ \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 = mgh \]

\[ h = \frac{1}{2mg} \left( m + \frac{1}{2} \right) v^2 = \frac{1}{2mg} (m + \frac{1}{2} m) v^2 \]

\[ h = \frac{3}{4} \frac{v^2}{g} \]

d) (5 pts) What is the acceleration as the disk goes up the ramp?

\[ \alpha = \frac{mg \sin \theta - \frac{I}{R^2} \alpha_m}{m - \frac{I}{R^2}} = \frac{2g \sin \theta}{m - \frac{I}{R^2}} \]

\[ \alpha_m = \frac{mg \sin \theta}{m - \frac{I}{R^2}} \]
Problem 4: A Bullet and a Block (30 points)

A bullet of mass $m$ and velocity $V_0$ plows into a block of wood at rest with mass $M$ which is part of a pendulum and stays inside the block. Assume that the acceleration due to gravity is $g$.

a) (5 pts) What is the velocity of the block/bullet pair after the collision?

b) (7 pts) How much work is done on the block during the collision?

c) (8 pts) How much energy is converted from mechanical to non-mechanical energy? (i.e., sound, heat etc.)

d) (10 pts) How high, $h$, does the block of wood go?

e) (5 pts) What is the period of oscillation?

\[
\begin{align*}
\Delta m &= (m+M) \quad \Delta V = \frac{v_m}{m+M} \\
W &= \frac{E_f - E_i}{m} = \frac{1}{2} (m+M) V_f^2 - \frac{1}{2} m V^2 = \frac{1}{2} \frac{m^2 V^2}{m+M} - \frac{1}{2} m V^2 \\
W &= -\frac{mM V^2}{2(m+M)} \\
W_{\text{net}} &= \Delta K = \frac{1}{2} (m+M) V_f^2 - 0 = \frac{1}{2} \frac{m^2 V^2}{m+M} \\
\frac{1}{2} (m+M) V_f^2 &= (m+M) g h \\
h &= \frac{V_f^2}{2g} = \frac{m^2 V^2}{2(m+M)^2} \\
T &= 2\pi \sqrt{\frac{L}{g}} 	ext{, independent of the masses.}
\end{align*}
\]
Part 5: (30 points)

You are a stunt artist on a motorbike jumps from a ramp to the top of a building. \( h \) is the vertical distance from the top of the ramp to the top of the building. The ramp is a distance \( d \) away from the building. The initial velocity is unknown, however the spectators notice that it just so happens that you reach the top of the building at the maximum height point during the flight, barely missing the building. In this problem you should ignore air friction. All your answers should be in terms of the variables given. The acceleration of gravity is \( g \) pointing down (\( g \) is a positive number).

a) (5 pts) What must be \( y \)-component of the initial velocity so that you just barely reach the top of the building?

b) (10 pts) How much time does it take for you to get there?

c) (15 pts) What is the magnitude and angle of the initial velocity? Again, to receive full, or even partial, credit, you must show your work that produces your answer.

\[ \begin{align*}
\text{a)} & \quad v_y - v_{0y} = d \\
& \quad v_{0y} = ? \\
& \quad v_y = 0 \\
& \quad a = -g \\
& \quad t = ?
\end{align*} \]

\[ \begin{align*}
v_y &= v_{0y} - gt \\
\Rightarrow \quad t &= \frac{v_{0y}}{g} = \frac{\sqrt{2gd}}{g} = \sqrt{\frac{2d}{g}} = t
\end{align*} \]

\[ \begin{align*}
v_x &= \frac{d}{t} = \frac{d}{\sqrt{\frac{a}{2d}}} = \frac{d\sqrt{2d}}{2} \\
\Rightarrow \quad v &= \sqrt{v_x^2 + v_y^2} = \sqrt{\frac{5}{2} gd}
\end{align*} \]

\[ \begin{align*}
\tan \theta &= \frac{v_y}{v_x} = 2 \Rightarrow \theta = \tan^{-1}(2) \\
\Rightarrow \quad \theta &= 63.4^\circ
\end{align*} \]
Problem 6: Moving in a platform (30 points)

You (mass = m) are standing at the edge of a platform with a radius R, mass M and I=\(\frac{1}{2}MR^2\). Neither you nor the platform is moving, initially and the platform can spin without friction. You need to get the platform moving with an angular speed \(\omega\). To do this you begin to walk at some constant, but unknown speed at the outer edge of the platform.

(16 points) At what speed, relative to the ground, do you need to be walking to get the platform moving with an angular speed \(\omega\)?

(14 points) How much work do you need to do to accomplish this?

\[a) \quad \text{angular momentum (total) is conserved so} \]
\[I\omega - mRV = 0 \Rightarrow V = \frac{I\omega}{mR} = \frac{\frac{1}{2}MR^2\omega}{mR} = \frac{1}{2}R\omega = V \]

\[b) \quad W = \Delta KE = \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2 - 0 = \]
\[= \frac{1}{2}m\left(\frac{1}{2}R\omega\right)^2 + \frac{1}{2} \cdot \frac{1}{2}MR^2\omega^2 = \]
\[= \left(\frac{m+M}{8}\right)MR^2\omega^2 = \frac{3}{8}mR^2\omega^2 = W \]

\[\text{if } m = M \]