Physics 218: Exam 3  
Sections 501 to 506, 522, 524, and 526.  

April 12th, 2013. 

Rules of the exam: 

1. You have the full class period to complete the exam.  
2. Formulae are provided on the last page. You may NOT use any other formula sheet.  
3. When calculating numerical values, be sure to keep track of units.  
4. You may use this exam or come up front for scratch paper.  
5. Be sure to put a box around your final answers and clearly indicate your work to your grader.  
6. Clearly erase any unwanted marks. No credit will be given if we can’t figure out which answer you are choosing, or which answer you want us to consider.  
7. Partial credit can be given only if your work is clearly explained and labeled.  
8. All work must be shown to get credit for the answer marked. If the answer marked does not obviously follow from the shown work, even if the answer is correct, you will not get credit for the answer. 

Sign below to indicate your understanding of the above rules. 

Name : ________________________________  
Student ID : ........................................ 
Signature : ........................................  
I want to pick the test in the class of (circle one): 2:20 pm or 5:30 pm
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Part 1: Equilibrium

A homogenous beam of mass=740 Kg is at one end supported by a cable that hangs from the ceiling as shown in the picture below. There is friction between the ground and the end of the beam.

(a) (8 points) Draw the free body diagram of the rod including relevant angles.

(b) (8 points) Find the tension in the cable. Express as a number with proper units.

Just write the usual equations of equilibrium

\[ \sum F, \hat{x} : f_s - T \cos(60) = 0 \]
\[ \sum F, \hat{y} : N - mg + T \sin(60) = 0 \]
\[ \sum \tau, \hat{z} : -mg \frac{L}{2} \cos(40) + LT \sin(60 - 40) = 0 \]

from the last equation we get

\[ T = \frac{mg \cos(40)}{2 \sin(20)} \Rightarrow T = 8121 \ N \]
(c) (8 points) Find the minimum coefficient of static friction $\mu_s$ between the beam and the floor for the beam to remain in this position. Express as a number with proper units.

From the previous set of equations we have

we need $f_s < \mu_s N \Rightarrow \mu_s > \frac{f_s}{N}$

from before $f_s - T \cos(60) = 0 \Rightarrow f_s = T \cos(60) = 4061 N$

from before $N - mg + T \sin(60) = 0 \Rightarrow N = mg - T \sin(60)$

$\Rightarrow N = 219 N$

therefore $\mu_s > \frac{f_s}{N} = \boxed{\mu_s > 18.5}$
Part 2: Physics of Billiards

In a billiard game the balls are solid spheres of radius $R$ and moment of inertia $I = \frac{2}{5} m R^2$.

(a) (10 points) Find the distance $h$ from the ground at which the cue should hit the ball if you want no initial friction to develop.

\[ \sum F_x : F = ma_x \]
\[ \tau_z : F(h - R) = I\alpha, \text{ using } a_x = -\alpha R \text{ we get } F(h - R) = -I \frac{a_x}{R} \]
\[ F(h - R) = -\frac{2}{5} m R^2 \frac{a_x}{R} = -\frac{2}{5} m R^2 \frac{F}{mR} = -\frac{2}{5} RF \]
\[ \Rightarrow h - R = \frac{2}{5} R \Rightarrow h = \frac{7}{5} R \]

(b) (10 points) Find the instantaneous angular acceleration of the ball if the cue hits the ball at the center ($h=R$) and the coefficient of kinetic friction between the ball and the table is $\mu_k$.

In this case the only force making a torque with respect to the center of mass is the friction.

\[ \tau_z : f_k R = I\alpha \Rightarrow \alpha = \frac{f_k R}{I} \]
\[ \alpha = \frac{\mu_k mg R}{\frac{2}{5} m R^2} \Rightarrow \alpha = \frac{5\mu_k g}{2R} \]
Part 3: Down the hill

A solid sphere of radius of \( R = 30\, \text{cm} \) and mass \( m = 2\, \text{Kg} \) is at the top of a hill of height \( h = 40\, \text{m} \). The sphere starts rolling down without slipping but \( 3/4 \) of the way down it passes through an oil deposit that removes all friction for the rest of its entire motion.

(a) (4 points) After passing the oil deposit is the angular momentum of the ball conserved? Explain why or why not.

Yes, because the sum of the external torques is zero.

(b) (10 points) Find the translational motion of the sphere when it reaches the valley at the bottom of the hill.

Calling \( h = 40\, \text{m} \) and \( h_1 = 10\, \text{m} \). In going from the top to the oil deposit we have a the force of static friction that does not work:

\[
mgh = mgh_1 + \frac{1}{2}mv_1^2 + \frac{1}{2}Iw_1^2
\]

\[
mgh_1 = \frac{1}{2}mv_1^2 + \frac{1}{2}I(v_1/R)^2 = v_1^2(m\frac{1}{2} + \frac{1}{2}5m) = v_1^2m\frac{7}{10}
\]

\[
\Rightarrow v_1^2 = \frac{10}{7}g(h-h_1) = 420\left(\frac{m}{s}\right)^2
\]

Now, from that point on angular velocity does not change, so considering conservation of energy from the point \( h_2 \) to the bottom we get:

\[
mgh_1 + \frac{1}{2}mv_1^2 + \frac{1}{2}Iw^2 = \frac{1}{2}mv^2 + \frac{1}{2}Iw^2
\]

\[
mgh_1 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv^2 \Rightarrow 2gh_1 + v_1^2 = v^2
\]

\[
v = \sqrt{2gh_1 + v_1^2} = \sqrt{196 + 420} = 24.8\left(\frac{m}{s}\right)
\]

(c) (10 points) Find how high up the other side of the valley will the rock go.

Simply when all the translational energy is spent on gravity:

\[
\frac{1}{2}mv^2 + \frac{1}{2}Iw^2 = mgh_f + \frac{1}{2}Iw^2
\]

\[
h_f = \frac{v^2}{2g} = 31.4\, \text{m}
\]
Part 4: Moving Squares

In a region outside any gravitational attraction a piece of homogenous steel of mass $3m$ has "L" shape and it moves towards the right at velocity $v$. Another small piece of homogenous steel has mass $m$, is square, and moves towards the first piece at velocity $2v$. At the moment of the collision both pieces stick together.

![Diagram of two pieces colliding]

(a) (4 points) Is the angular momentum of the two-piece system conserved? Explain why or why not.

Yes, it is conserved b/c the sum of the external torques is zero.

(b) (4 points) Is the linear momentum of the two-piece system conserved? Explain why or why not.

Yes, it is conserved b/c the sum of the external forces is zero.

(c) (4 points) Is the mechanical energy of the two-piece system conserved? Explain why or why not.

No, because the work of other forces in the collision might not be zero.

(d) (4 points) Put a coordinate system and find the vertical component ($y$-component) of the position of the center of mass of the L-shaped piece. [Hint: imagine the L-shape piece as three smaller blocks]

$$r_{cm,y} = \frac{\sum m_i y_i}{\sum m_i} = \frac{-m\frac{L}{2} + 2m\frac{L}{2}}{3m} = \frac{L}{6}$$
(e) (4 points) Find the horizontal component (x-component) of the velocity of the center of mass of the two-piece system.

\[
v_{cm,x} = \frac{\sum m_i v_i}{\sum m_i} = \frac{3mv - m2v}{4m} = \frac{v}{4}
\]

(f) (8 points) After the pieces are stuck together find the angular velocity of rotation of the now whole square around its center of mass.

Using conservation of angular momentum in the \( \hat{z} \) direction around the center of the big square we get:

\[
L_i = L_f = 3mvL \frac{6}{6} + m2vL \frac{2}{2} = \frac{1}{4}mL^2 \omega_f
\]

\[
\frac{3}{2}mvL = \frac{4}{6}mL^2 \omega_f
\]

\[
\omega_f = \frac{9v}{4L}
\]

(g) (10 points (bonus)) What is the total rotational and translational kinetic energy before and after the collision?

\[
R_i + K_i = \frac{1}{2}3mv^2 + \frac{1}{2}m4v^2 = \frac{7}{2}mv^2 = \frac{56}{16}mv^2 =
\]

\[
R_f + K_f = \frac{1}{2}4mv^2_{cm} + \frac{1}{2}I\omega_f^2
\]

\[
= \frac{1}{2}4m(v)^2 + \frac{1}{2}6mL^2(\frac{9v}{4L})^2
\]

\[
= \frac{1}{8}mv^2 + \frac{27}{16}mv^2 = \frac{29}{16}mv^2
\]
Part 5: Can of water

A can of radius $R$ rotates freely with initial angular velocity $\omega_0$. The can has moment of inertia $I_c$ around its rotating axis. As the can is rotating somebody opens a faucet and a dense liquid starts dropping in the can, for a total mass of liquid of $m_l$. Assume the can rotates so slowly that the liquid fills the can homogenously (i.e. the level of water is the same for all radius even though the can is rotating).

(a) (8 points) Assuming the liquid in the can is moving like a solid disk of radius $R$ find the angular velocity $\omega_f$ of the can filled with the liquid.

A the water drops in it exerts no torque along the direction of the free-rotating axis. Since there are no torques around the free rotating axis the angular momentum is conserved.

\[ L_i = L_f \]
\[ I_c \omega_0 = (I_c + \frac{1}{2} m_l R^2) \omega_f \]
\[ \omega_f = \frac{I_c \omega_0}{I_c + \frac{1}{2} m_l R^2} \]

(b) (8 points) After being filled the can develops a hole in the outer edge and the liquid starts going out. When the can is empty again, what is the angular velocity ? If you think it did not change explain why ?

The angular velocity is still $\omega_f$, it did not change. As the liquid goes out there is still no torque of forces on the can and therefore no angular acceleration on it. Thus the angular velocity remains constant.

(c) (8 points) When the can is again empty due to the hole in the can, what is the new angular momentum of the empty can?

As the liquid goes out it removes angular momentum. When the can is again empty the angular momentum is simply \( L = I_c \omega_f \).