Physics 218 Exam 3
Spring 2011, Sections 513,514,515,526,528

Do not fill out the information below until instructed to do so!

Name: _______ SOLUTIONS ______
Signature: ______________________
Student ID: _____________________
E-mail: _________________________
Section #: _______________________

Rules of the exam:
1. You have the full class period to complete the exam.
2. When calculating numerical values, be sure to keep track of units.
3. You may use this exam or come up front for scratch paper.
4. Be sure to put a box around your final answers and clearly indicate your work to your grader.
5. Clearly erase any unwanted marks. No credit will be given if we can’t figure out which answer you are choosing, or which answer you want us to consider.
6. Partial credit can be given only if your work is clearly explained and labeled.
7. All work must be shown to get credit for the answer marked. If the answer marked does not obviously follow from the shown work, even if the answer is correct, you will not get credit for the answer.

Put your initials here after reading the above instructions: _______
Part 1: (25p) Momentum and Collisions

Problem 1.1: Two hockey pucks (disk-shaped) of radii $R$ and mass $m$ are moving towards each other in a surface without friction with equal and opposite velocity $v$ on a direct collision course. Both pucks are also rotating counter-clockwise around their respective center of masses with angular velocity $\omega$ as shown in the picture below.

Question 1.1.1: (2p) Compute the velocity of the center of mass of the system of two pucks?

$$v_{CM} = \frac{\sum m_i v_i}{\sum m_i} = mv - mv = 0$$

Question 1.1.2: (3p) What is the angular momentum of the system of two pucks?

$$L = 2I\omega_2 = 2 \frac{1}{2} mR^2 \omega_2 = mR^2 \omega_2$$

Question 1.1.3: (5p) Would the angular momentum be conserved before and after the collision? Explain your answer.

Yes. Before, during and after the collision the sum of the external torques on the two-puck system is zero and as a consequence the angular momentum should be conserved.

Question 1.1.4: (10p) At the moment the pucks collide each other they immediately stick together becoming a solid two-puck system. What is the moment of inertia of the two-puck system with respect to its own center of mass?

$$I_{two-puck\ cm} = 2I_{disk\ at\ edge}$$

$$I_{disk\ at\ edge} = I_{disk\ at\ center} + mR^2 = \frac{1}{2} mR^2 + mR^2 = \frac{3}{2} mR^2 \Rightarrow I_{two-puck\ cm} = 3mR^2$$

Question 1.1.5: (5p) Would the new two-puck system spin around its CM axis? If not explain why; if yes compute the angular velocity.

Yes, it will. Since angular momentum is conserved then we obtain:

$$mR^2 \omega = I_{two-puck\ cm} \omega_f \Rightarrow \omega_f = \frac{mR^2 \omega}{3mR^2} = \frac{\omega}{3}$$
Part 2: (25p) Dynamics of Rotational Motion

Problem 2.1: A disk of mass $m_d$ and radius $R$ and a block of mass $m_b$ are falling down in an inclined plane and angle $\alpha$ above the horizontal. The axis of the disk is connected with a solid massless rod to the block at a height $R$ as shown in the figure. As they fall down the incline the disk rolls without slipping and the block slips without rolling and without friction.

Question 2.1.1: (5p) Put a coordinate system, and draw the free body diagram of the disk and the block, including relevant angles.

Question 2.1.2: (10p) Write ALL equations of motion that would allow you to solve the whole problem.

Disk:

- $y$: $N_d - m_d g \cos \alpha = 0 \Rightarrow N_d = m_d g \cos \alpha$
- $x$: $T - f_d + m_d g \sin \alpha = m_d a_x$
- $z$: $- f_d R = I \alpha, \text{with } a_x = -R\alpha \Rightarrow f_d = \frac{I \alpha_x}{R^2}$

Block:

- $y$: $N_b - m_b g \cos \alpha = 0 \Rightarrow N_b = m_b g \cos \alpha$
- $x$: $m_b g \sin \alpha - T = m_b a_x$

Question 2.1.3: (10p) Solve the equations and find the linear acceleration of the disk (or block) in the direction of the inclined plane.

Plug $f_d$ into the $x$ equations of the disk and block:

Disk $x$: $T - \frac{I \alpha_x}{R^2} + m_d g \sin \alpha = m_d a_x$

Block $x$: $m_b g \sin \alpha - T = m_b a_x$

These are two equations with two unknowns. Find $T$ from one and plug into the other

Block $x$: $m_b g \sin \alpha - m_b a_x - \frac{I \alpha_x}{R^2} + m_d g \sin \alpha = m_d a_x$

$\Rightarrow (m_b + m_d)g \sin \alpha = \left(m_b + m_d + \frac{I}{R^2}\right)a_x \Rightarrow a_x = \frac{(m_b + m_d)g \sin \alpha}{(m_b + m_d + \frac{I}{R^2})}$

Furthermore since $I = \frac{m_d R^2}{2}$ we obtain $a_x = \frac{(m_b + m_d)}{(m_b + \frac{3m_d}{2})}g \sin \alpha$
Part 3: (25p) Energy of Rotational Motion

Problem 3.1: A small bicycle is made of a front tire of mass $m_2$, radius $R_2$ and moment of inertia $I_2$ and a rear tire of mass $m_1$, radius $R_1$ and moment of inertia $I_1$. The motorcycle is initially at rest on an inclined plane at a height $h$ and as the distance between the tires is $d$ as shown in the diagram below. The tires can be considered disks and they roll without slipping.

Question 3.1.1: (2p) Will the energy be conserved as the bicycle rolls down? Explain your reasoning?

Yes, the energy is conserved as the work of non-conservative forces is zero. The only non-conservative force is the static friction that simply does not work.

Question 3.1.2: (20p) Using conservation of energy find the velocity of the bicycle when the front door reaches the bottom of the incline.

Let’s call $h_1$ the height difference between the center of $m_1$ and $m_2$.

\[ E_i = m_1 g (h + h_1) + m_2 g h \]
\[ E_f = m_1 g h_1 + \frac{1}{2} m_1 v^2 + \frac{1}{2} l_1 \omega_1^2 + \frac{1}{2} m_2 v^2 + \frac{1}{2} l_2 \omega_2^2 \]

Replacing $\omega_1 = \frac{v}{R_1}$ and $\omega_2 = \frac{v}{R_2}$ and equating $E_i = E_f$, we obtain

\[ m_1 g h_1 + m_2 g h = \frac{1}{2} v^2 \left( m_1 + m_2 + \frac{l_1}{R_1^2} + \frac{l_2}{R_2^2} \right) \]

from which we get

\[ v^2 = \frac{2 (m_1 + m_2) g h}{m_1 + m_2 + \frac{l_1}{R_1^2} + \frac{l_2}{R_2^2}} \]

Question 3.1.3: (3p) What would that speed be if $l_1 = \frac{m_1 R_1^2}{2}$ and $l_2 = \frac{m_2 R_2^2}{2}$?

\[ v^2 = \frac{2 (m_1 + m_2) g h}{m_1 + m_2 + \frac{m_1}{2} + \frac{m_2}{2}} = \frac{2 (m_1 + m_2) g h}{\frac{3}{2} (m_1 + m_2)} = \frac{4}{3} g h \]
Part 4: (25p) Static Equilibrium

Problem 4.1: A ring of radius $R$ and mass $m_R$ can freely rotate around a nail passing in its rim and attached to a wall as shown on the Figure A below. A block of mass $m_b$ is hanging from a string which is also attached to the nail, so that the string forces the ring to move until equilibrium is reach as shown in Figure B.

Question 4.1.1: (5p) Draw a free body diagram of the system at equilibrium (Figure B).

Question 4.1.2: (17p) Find the angle $\alpha$ the ring moved to reach equilibrium.

From the diagram above it is clear that equilibrium is reach when the sum of all torques with respect to the nail is zero. Requiring that we get:

$$g m_R R \sin \alpha - g m_b (R - R \sin \alpha) = 0 \quad \Rightarrow \quad m_R \sin \alpha - m_b (1 - \sin \alpha) = 0$$

$$\Rightarrow \quad \sin \alpha (m_R + m_b) = m_b \quad \Rightarrow \quad \sin \alpha = \frac{m_b}{(m_R + m_b)}$$

Question 4.1.3: (3p) Find the angle in degrees if $m_b = 2m_R$.

From the previous equation we get $\sin \alpha = \frac{m_b}{(m_R + m_b)} = \frac{2m_R}{(m_R + 2m_R)} = \frac{2}{3} \Rightarrow \alpha = 41.8^\circ$