Physics 218: Exam 1
Sections 501 to 506, 522, 524, and 526.

February 15th, 2013.

Rules of the exam:

1. You have the full class period to complete the exam.

2. Formulae are provided on the last page. You may NOT use any other formula sheet.

3. When calculating numerical values, be sure to keep track of units.

4. You may use this exam or come up front for scratch paper.

5. Be sure to put a box around your final answers and clearly indicate your work to your grader.

6. Clearly erase any unwanted marks. No credit will be given if we can't figure out which answer you are choosing, or which answer you want us to consider.

7. Partial credit can be given only if your work is clearly explained and labeled.

8. All work must be shown to get credit for the answer marked. If the answer marked does not obviously follow from the shown work, even if the answer is correct, you will not get credit for the answer.

Sign below to indicate your understanding of the above rules.

Name : ____________________________

Student ID : ................................

Signature : .................................
<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basics</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>Position, Velocity, and Acceleration</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>The Movement of the Earth, in perspective</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>Racing</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>Landing on Mars</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td><strong>Total:</strong></td>
<td><strong>120</strong></td>
<td><strong>120</strong></td>
</tr>
</tbody>
</table>

**Part 1: Basics**

(17 points)

Solve the following problems in basic unit conversions.

(a) (5 points) What are the basic units of time, mass and length in the International System of units?

For time is seconds, for mass is kg, and for length is meter.

(b) (5 points) A common unit of pressure in the US is the psi or \( \text{pounds} \text{ inch}^{-2} \). Knowing that a pound is 4.45 Newtons and that an inch is 0.0254 m express 10 psi in units of \( \frac{\text{Newton}}{\text{m}^2} \).

Knowing that: \( 1 \text{ pound} = 4.45 \text{ Newtons} \)

\( 1 \text{ inch} = 0.0254 \text{ m} \)

We get: \( 10 \frac{\text{pounds}}{\text{in}^2} = 10 \times 4.45 \frac{\text{Newton}}{(0.0254 \text{ m})^2} = 68975 \frac{\text{Newton}}{\text{m}^2} \)

(c) (7 points) Compute the quantity \( \vec{c} = (\vec{a} - \vec{b})(\vec{a} \cdot \vec{b}) \), where the vector \( \vec{a} \) has a magnitude of 4 straight in the \( \hat{y} \) direction, and the vector \( \vec{b} \) has a magnitude of 5 at an angle of 37° North of the \( \hat{x} \) direction.

\[
\vec{a} = (0, 4) \\
\vec{b} = (5\cos(37), 5\sin(37)) \\
\vec{a} \cdot \vec{b} = 20\sin(37) \approx 12.04 \\
\vec{a} - \vec{b} = (-5\cos(37), 4 - 5\sin(37)) \approx (-4, 1) \\
\vec{c} = 20\sin(37)(-5\cos(37), 4 - 5\sin(37)) \approx (-48, 12)
\]
**Part 2: Position, Velocity, and Acceleration**  
(25 points)

An object is moving in the \((x,y)\) plane. The two plots below show the \(x\) and \(y\) position of the object as a function of time.

(a) (4 points) Estimate the distance of the object to the origin of the coordinate system when \(t = 4\) s.

\[
\begin{align*}
\text{At } t = 4\text{s we get : } x(t = 4\text{s}) &= 2 \text{ m} \\
\text{At } t = 4\text{s we get : } y(t = 4\text{s}) &= 2 \text{ m} \\
\text{from which we get : } |\vec{x}| &= \sqrt{2^2 + 2^2} \approx 2.82 \text{ m}
\end{align*}
\]

(b) (5 points) Estimate the magnitude of the object’s velocity at time \(t = 2\) s.

\[
\begin{align*}
\text{From the slope of the first plot at } t = 2\text{s we get : } v_x(t = 2\text{s}) &= -2 \frac{m}{s} \\
\text{From the slope of the second plot at } t = 2\text{s we get : } v_y(t = 2\text{s}) &= 0.5 \frac{m}{s} \\
\text{from which we get : } |\vec{v}| &= \sqrt{(-2)^2 + 0.5^2} = 2.061 \frac{m}{s}
\end{align*}
\]

(c) (4 points) If \(y\) represents North and \(x\) represents East, what direction is the object moving at \(t = 3\) s? Explain your reasoning.

\[
\begin{align*}
\text{From the slope of the first plot at } t = 3\text{s we get : } v_x(t = 3\text{s}) &= 0 \frac{m}{s} \\
\text{From the slope of the second plot at } t = 3\text{s we get : } v_y(t = 3\text{s}) &= 0 \frac{m}{s} \\
\text{from which we get that the object is moving in the North direction.}
\end{align*}
\]
(d) (6 points) Given the following plot of y-position on the left what curve in the right most plot most accurately depicts the way the velocity in the y component behaves? **Explain your reasoning**!

![Plot of y-position and velocity](image)

From the left plot we can see that $v_y$ is going down in the range $0 < t < 1 \text{s}$, it is constant in the range $1 < t < 3$, exactly zero at $t = 3.5 \text{s}$ and negative after $t = 3.5 \text{s}$. The only curve satisfying all these is D.

(e) (6 points) Given the following plot of x-position on the left what curve in the right most plot depicts the acceleration in the x component? **Explain your reasoning**!

![Plot of x-position and acceleration](image)

From the left plot we can see that the $a_x$ is negative at $t = 1 \text{ seconds}$, zero at about $t = 2 \text{ seconds}$ and positive at $t = 3 \text{ seconds}$. The only curve in the right hand plot that satisfies that is curve E.
Part 3: The Movement of the Earth, in perspective (26 points)

(a) Consider the earth a perfect sphere with radius $R = 6,400$ kilometers that rotates around its axis with a period of 24 hours.

i. (6 points) The gravitational constant in earth varies from about $g=9.78 \, \text{m} \, \text{s}^{-2}$ at the equator to $9.82 \, \text{m} \, \text{s}^{-2}$ at the poles. Compute the acceleration in $\text{m} \, \text{s}^{-2}$ of an object fixed in the equator due to the rotation of the earth. Is the difference in $g$'s mostly explained by the rotation of the earth around its axis or mostly by something else?

\[
|\vec{a}_\perp| = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2 \times 6.4 \times 10^6 \, \text{m}}{86400^2 \, \text{s}^2} = 0.0338 \, \text{m} \, \text{s}^{-2}
\]

so rotation explains most of the $0.04 \, \text{m} \, \text{s}^{-2}$ difference in $g$'s

ii. (6 points) Compute the velocity in $\text{m} \, \text{s}^{-1}$ of an object fixed in College Station due to the rotation of the earth knowing that the latitude of College Station is $\phi = 30.62^\circ$ as shown in the graph below.

(b) Assuming that the earth moves in a uniform circular motion around the sun, and knowing that the distance from the earth to the sun is about $1.5 \times 10^{11}$ meters and that there are $3.15 \times 10^7$ seconds in a year.

i. (4 points) Find the orbital speed of the earth around the sun in $\text{m} \, \text{s}^{-1}$.

\[
|\vec{v}_0| = \frac{2\pi R}{T} = \frac{2\pi \times 6.4 \times 10^6 \times \cos(30.62^\circ) \, \text{m}}{86400 \, \text{s}} \\
|\vec{v}_0| \approx 400 \, \text{m} \, \text{s}^{-1}
\]
ii. (4 points) Compute the acceleration of the earth around the sun in units of \(\frac{m}{s^2}\). Is the acceleration parallel or transverse to the earth orbital velocity? What direction does it point to?

\[
|a_\perp| = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2 \times 1.5 \times 10^{11} m}{\pi^2 \times 9.92 \times 10^{14} s^2} = 5.96 \times 10^{-3} \frac{m}{s^2}
\]

It is transverse to the orbital velocity of the earth, pointing towards the sun.

(c) (6 points) The sun and all solar planets rotate around the center of the galaxy in an approximate uniform circular motion with an orbital velocity of about 220 \(\frac{km}{s}\). Knowing that the distance between the sun and the center of the galaxy is about 8500 parsecs (1 parsec=3.09 \times 10^{16} meters) find the time it would take our sun and its system to make a full rotation around the galaxy. **Express in millions of years.**

\[
|\vec{v}| = \frac{2\pi R}{T} \Rightarrow T = \frac{2\pi R}{|\vec{v}|} = \frac{2\pi \times 2.62 \times 10^{20} m}{2.2 \times 10^5 \frac{m}{s^2}} = 7.5 \times 10^{15} s
\]

Dividing by \((3600 \times 24 \times 365)\) we get \(T=237\) million years.
Part 4: Racing

Cars A and B are racing against each other in a long straight freeway. At time $t = 0$ both cars are in the same position and have velocities $v_A$ and $v_B$ and accelerations $a_A$ and $a_B$, all positive. It is also noted that car A has larger speed than car B and that car A has a smaller acceleration than car B. Both cars keep a constant acceleration.

(a) (5 points) Write an expression that will tell you the distance between car A and car B as a function of time, $d(t) = \text{position}_{\text{car A}} - \text{position}_{\text{car B}}$.

\[ d(t) = x_A(t) - x_B(t) = (v_A - v_B)t + \frac{1}{2}(a_A - a_B)t^2 \]

(b) (6 points) Schematically graph the $d(t)$ distance as a function of time below. [Hint: consider the signs of the different terms]

(c) (6 points) Find the time(s) in which both cars are at the same position.

They are at the same position for times when $d(t) = 0$

\[ d(t) = (v_A - v_B)t + \frac{1}{2}(a_A - a_B)t^2 = t \left( v_A - v_B + \frac{a_A - a_B}{2} \right) \]

from which we get that $d(t) = 0$ when either $t = 0$ or $t = \frac{2(v_A - v_B)}{a_B - a_A}$
(d) (6 points) If the race lasted forever, what is the maximum distance car A will ever be ahead of car B?

When $d(t)$ takes the most positive value is when car A is the maximum distance ahead. This happens at $t = t_m$ when the derivative of $d(t)$ is zero.

\[
\frac{d}{dt} d(t_m) = (v_A - v_B) + (a_A - a_B)t_m = 0 \Rightarrow t_m = \frac{v_A - v_B}{a_B - a_A}
\]

\[
d(t_m) = (v_A - v_B) \frac{v_A - v_B}{a_B - a_A} + \frac{a_A - a_B}{2} \left( \frac{v_A - v_B}{a_B - a_A} \right)^2
\]

\[
d(t_m) = \frac{(v_A - v_B)^2}{a_B - a_A} - \frac{1}{2} \frac{(v_A - v_B)^2}{a_B - a_A} = \frac{1}{2} \frac{(v_A - v_B)^2}{a_B - a_A}
\]

(e) (5 points) If the race lasted forever, what is the maximum distance car B will ever be ahead of car A?

When $d$ takes the most negative value is when car B is the maximum distance ahead. However, because $v_B > v_A$ and $a_A > a_B$ the equation find in the first question shows that the distance becomes more and more negative with time. Hence the more time it elapses the more distance ahead is car B. The answer is thus infinite.
Part 5: Landing on Mars

The landing sequence of the mars rover is depicted below and requires the use of a rocket system that you need to design so that it deposits the rover onto martian soil as smoothly as possible. The landing sequence requires that your rocket will fire its engines providing a constant deceleration when the rover is at 1 km height and falling vertically at a speed of 80 m/s.

(a) (6 points) Find the acceleration the rocket (and rover) need to have such that the rover is deposited smoothly on the ground.

Knowing that $v_y = 0$ at the moment of the touchdown we get:

$$v_y^2 = v_{y,0}^2 + 2a_y(y_f - y_0) \Rightarrow a_y = \frac{v_y^2 - v_{y,0}^2}{2(y_f - y_0)} \quad (1)$$

$$a_y = \frac{-(-80 \text{ m/s})^2}{-2 \times 10^2 \text{ m}} = +3.2 \frac{\text{m}}{\text{s}^2} \quad (2)$$

(b) (6 points) Find the time elapsed from the moment the rocket fired its engines to the moment the rover touches the ground.

$$v_{y,0} + a_y t_g = 0 \Rightarrow t_g = \frac{-v_{y,0}}{a_y} = 25 \text{ s} \quad (3)$$
(c) (6 points) Imagine the altitude was mismeasured and the engines are fired when the rover was really 0.98 km above ground and the falling speed still 80 $\frac{m}{s}$ . If the engine still results in the acceleration found in the first point, find the vertical velocity of the rover at the moment of touchdown.

The two basic conditions are

$$v_{y,f}^2 = v_{y,0}^2 + 2a_y(y_f - y_0) = \left(-80 \frac{m}{s}\right)^2 - 2 \cdot 3.2 \frac{m}{s^2} \cdot 980 m$$  \hspace{1cm} (4)

$$\Rightarrow v_{y,f} = 11.3 \frac{m}{s}$$ \hspace{1cm} (5)

(d) (6 points) After leaving the rover in the ground the rocket quickly tilt itselfs at an angle of 30° with respect to the horizontal and flies away with the same acceleration magnitude for 10 more seconds. Find the horizontal distance the rocket is from the rover when the 10 seconds have elapsed.

$$x(t) = \frac{1}{2} \cdot 3.2 \cdot \cos(30) \frac{m}{s^2} t^2$$ \hspace{1cm} (6)

$$x(t = 10s) \approx 138 m$$ \hspace{1cm} (7)
Formula Sheet: Exam I

Mathematical Formulae:

\[ \vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z} \]
\[ |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} \]
\[ \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z \]
\[ = |\vec{A}||\vec{B}| \cos(\gamma \text{between } \vec{A} \text{ and } \vec{B}) \]
\[ at^2 + bt + c = 0 \Rightarrow t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
if \( x(t) = at^n \Rightarrow \frac{dx}{dt} = n a t^{n-1} \)
\[ \int_{t_1}^{t_2} a t^n dt = \frac{a}{n + 1} (t_2^{n+1} - t_1^{n+1}) \]

Always true:

\[ \vec{v} = \frac{d\vec{x}}{dt} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2} \]
\[ \vec{v}_{AVG} = \frac{\vec{x}_f - \vec{x}_i}{t_f - t_i}, \quad \vec{a}_{AVG} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} \]
\[ \vec{x}(t) = \vec{x}_0 + \int_0^t \vec{v}(t)dt \]
\[ \vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}(t)dt \]

Circular Motion :

\[ \vec{a}_\perp = \frac{|\vec{v}|^2}{R} \]
\[ \vec{a}_\parallel = \frac{d|\vec{v}|}{dt} \]
\[ f = \frac{1}{T}, \quad \omega = \frac{2\pi}{T} \]

Relative Movement:

\[ \vec{x}_{P/A} = \vec{x}_{P/B} + \vec{x}_{B/A} \]
\[ \vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A} \]

Uniform Circular Motion :

\[ |\vec{v}| = \frac{2\pi R}{T} \]
\[ \vec{a}_\perp = \frac{|\vec{v}|^2}{R} = \frac{4\pi^2 R}{T^2} \]
\[ \vec{a}_\parallel = 0 \]