Physics 218: Exam 1  
Class of 2:20pm

February 14th, 2012.

Rules of the exam:

1. You have the full class period to complete the exam.

2. Formulae are provided on the last page. You may NOT use any other formula sheet.

3. When calculating numerical values, be sure to keep track of units.

4. You may use this exam or come up front for scratch paper.

5. Be sure to put a box around your final answers and clearly indicate your work to your grader.

6. Clearly erase any unwanted marks. No credit will be given if we can’t figure out which answer you are choosing, or which answer you want us to consider.

7. Partial credit can be given only if your work is clearly explained and labeled.

8. All work must be shown to get credit for the answer marked. If the answer marked does not obviously follow from the shown work, even if the answer is correct, you will not get credit for the answer.

Sign below to indicate your understanding of the above rules.

Name : __________________________

Student ID : ______________________

Signature : _______________________
<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basics</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Velocity and Acceleration</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Aircraft Carrier</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>Acceleration in both components</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>Reading DVD’s</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Aliens</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Total:</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

**Part 1: Basics**

(12 points)

Solve the following problems in basic unit conversions.

(a) (2 points) What are the basic units of time, mass and length in the International System of units?

For time is seconds, for mass is kg, and for length is meter.

(b) (5 points) Convert \(56 \text{ grams/cm}^3\) to \(\frac{\text{kg}}{\text{m}^3}\)

Knowing that:

\[10^3 \text{grams} = 1 \text{kg} \Rightarrow 1 \text{grams} = 10^{-3} \text{kg}\]
\[10^2 \text{cm} = 1 \text{m} \Rightarrow 1 \text{cm} = 10^{-2} \text{m}\]

We get:

\[
56 \text{ grams/cm}^3 = 56 \times 10^{-3} \frac{\text{kg}}{(10^{-2} \text{m})^3} = 56 \times 10^{-3} \frac{\text{kg}}{10^{-6} \text{m}^3} = 56 \times 10^3 \frac{\text{kg}}{\text{m}^3}
\]

(c) (5 points) Compute the magnitude of the vector \(\vec{c} = \vec{a} - \vec{b}\), where the vector \(\vec{a}\) has a magnitude of \(3 \frac{\text{m}}{\text{s}^2}\) straight in the North direction, and the vector \(\vec{b}\) has a magnitude of \(5 \frac{\text{m}}{\text{s}^2}\) at an angle of 37° North of the East direction.

\[
\vec{a} = (0, 3) \frac{\text{m}}{\text{s}^2}
\]
\[
\vec{b} = (5\cos(37), 5\sin(37)) \frac{\text{m}}{\text{s}^2}
\]
\[
\vec{c} = \vec{a} - \vec{b} = (-5\cos(37), 3 - 5\sin(37)) \frac{\text{m}}{\text{s}^2}
\]
\[
|\vec{c}| = \sqrt{(-5\cos(37))^2 + (3 - 5\sin(37))^2} \approx 4 \frac{\text{m}}{\text{s}^2}
\]
Part 2: Velocity and Acceleration

An object is moving in the (x, y) plane. The two plots below show the x and y position of the object as a function of time.

(a) (4 points) Estimate the magnitude of the object’s velocity at time $t = 2s$.

From the slope of the first plot at $t = 2s$ we get: $v_x(t = 2s) = -1 \frac{m}{s}$

From the slope of the second plot at $t = 2s$ we get: $v_y(t = 2s) = 0.5 \frac{m}{s}$

from which we get: $|\vec{v}| = \sqrt{(-1)^2 + 0.5^2} = 1.118 \frac{m}{s}$

(b) (4 points) If y represents North and x represents East, what direction is the object moving at $t = 4s$? Explain your reasoning.

From the slope of the first plot at $t = 4s$ we get: $v_x(t = 4s) = 0 \frac{m}{s}$

From the slope of the second plot at $t = 4s$ we get: $v_y(t = 4s) < 0 \frac{m}{s}$

from which we get that the object is moving in the South direction.

(c) (4 points) At time $t = 4s$ determine the sign of the acceleration in the x and y components.

From the concavity of the first plot at $t = 4s$ we get: $a_x(t = 4s) > 0 \frac{m}{s}$

From the concavity of the second plot at $t = 4s$ we get: $a_y(t = 4s) < 0 \frac{m}{s}$
Part 3: Aircraft Carrier

An AV-8B II plane needs to land on its aircraft carrier. As it is approaching the landing strip the plane is heading south at a speed of 50 \( \frac{m}{s} \) with respect to the air. The carrier measures the speed of the air to be 30 \( \frac{m}{s} \) at a direction of 30 degrees west of north with respect to the carrier.

(a) (4 points) Draw a basic diagram indicating the relevant reference frames and draw ALL relevant velocities.

\[ \vec{v}_{P/C} = \vec{v}_{P/A} + \vec{v}_{A/C} = (0, -50) \frac{m}{s} + (-30 \sin(30^\circ), 30 \cos(30^\circ)) \frac{m}{s} \]
\[ = (-30 \sin(30^\circ), -50 + 30 \cos(30^\circ)) \frac{m}{s} \]
\[ \vec{v}_{P/C} = (-15, -24) \frac{m}{s} \]

(b) (6 points) Compute both components of the velocity of the plane with respect to the carrier

\[ |\vec{v}_{P/C}| = \sqrt{(-30 \sin(30^\circ))^2 + (-50 + 30 \cos(30^\circ))^2} = 28.3 \frac{m}{s} \]
so the plane will not land successfully.

(c) (4 points) If the only condition for a successful landing is that the speed of the plane at the moment of the landing is less than 25 \( \frac{m}{s} \) with respect to the carrier ship, will the plane land successfully?

(d) (4 points) If before landing the direction of the wind changes to be now in the north direction while its speed remains at 30 \( \frac{m}{s} \) as before, will the plane be able to land now?

\[ |\vec{v}_{P/C}| = \sqrt{-20^2} = 20 \frac{m}{s} \]
so the plane will land successfully.
Part 4: Acceleration in both components (24 points)

On the surface of a moon with magnitude of gravity $g_m$ a scientific device is launched at an angle $\alpha$ with an initial speed of $v_0$. The device has a set of burners on its side providing the device with a constant horizontal acceleration in the positive $x$ direction of $a_x$. Assume the mass of the device is constant and the atmosphere of the moon can be neglected. All answers must be in terms of the known variables $g_m$, $\alpha$, $v_0$, and $a_x$.

(a) (4 points) Draw the problem and indicate a coordinate system

\[ \overrightarrow{g_m} \]

(b) (5 points) Write the equations of motion of the device for that coordinate system. That is the horizontal and vertical components as a function of time, $X(t)$ and $Y(t)$. If you put terms that are zero indicate so.

\[ x(t) = v_0 \cos(\alpha) t + a_x \frac{t^2}{2} \]
\[ y(t) = v_0 \sin(\alpha) t - g_m \frac{t^2}{2} \]

(c) (5 points) Find the time it takes the device to fall to the ground.

\[ y(t_g) = 0 = v_0 \sin(\alpha) t_g - g_m \frac{t_g^2}{2} \]
\[ t_g = \frac{2v_0 \sin(\alpha)}{g_m} \]
(d) (5 points) Find the range of the device, this is the horizontal distance from where it launched to where it landed.

\[ x(t_g) = v_0 \cos(\alpha)t_g + a_x \frac{t_g^2}{2} \]

Plugging \( t_g \) from the previous points we get

\[ x(t_g) = \frac{2v_0^2}{g_m} \left[ \cos(\alpha)\sin(\alpha) + \frac{a_x}{g_m} \sin^2(\alpha) \right] \]

(e) (5 points) Find the horizontal position of the device at the point it reaches the maximum height. If you make any assumption justify it.

\[ v_y(t_m) = 0 = v_y\sin(\alpha) - g_m t_m \Rightarrow t_m = \frac{v_y\sin(\alpha)}{g_m} \]

\[ x(t_m) = v_0 \cos(\alpha)t_m + a_x \frac{t_m^2}{2} \]

\[ = v_0 \cos(\alpha) \frac{v_y\sin(\alpha)}{g_m} + a_x \frac{v_y^2 \sin^2(\alpha)}{2g_m^2} \]

\[ x(t_m) = \frac{v_0^2}{g_m} \left[ \cos(\alpha)\sin(\alpha) + \frac{a_x}{2g_m} \sin^2(\alpha) \right] \]
Part 5: Reading DVD’s  (20 points)
A DVD disk is read by spinning the disk over a laser system whose radial position from the center of the disk can be controlled from a radius of 2.2cm and up to a radius of 5.7cm. However, in a DVD the frequency of revolution must be controlled such that the linear velocity of the disk at the radius where the laser is positioned must always be a constant \(3.5 \text{ m/s}\). Uniform circular motion can be assumed in this problem. All answers must be a number with proper units.

(a) (6 points) Find the frequencies at which the DVD must spin when the laser system is reading at its inner radius and at its outer radius. Express in units of revolutions per second.

\[
v = \frac{2\pi R_L}{T_{dvd}} = 2\pi R_L f_{dvd} = 3.5 \text{ m/s} \Rightarrow f_{dvd} = \frac{3.5 \text{ m/s}}{2\pi R_L}
\]

at inner radius: \(\frac{3.5 \text{ m/s}}{2\pi 0.022m} = 25.3 \text{ revolutions/second}\)

at outer radius: \(\frac{3.5 \text{ m/s}}{2\pi 0.057m} = 9.8 \text{ revolutions/second}\)
(b) (4 points) Find the acceleration at the outer radius of the disk when the disk is spinning at the two frequencies find in the previous problem. Express in units of \( \frac{m}{s^2} \).

\[
a = \frac{4\pi^2 \cdot 5.7 \text{ cm}}{T_{\text{dvd}}^2} = 4\pi^2 \cdot 0.057 \cdot m \cdot f_{\text{dvd}}^2
\]

at inner radius \( a_{\text{inner}} = 1440 \frac{m}{s^2} \)

at outer radius \( a_{\text{outer}} = 216 \frac{m}{s^2} \)

(c) (10 points) Now a little ant hops into the DVD and sits at radius \( R_{\text{ant}} = 3 \text{ cm} \). If the laser system is reading at radius \( R_L = 4 \text{ cm} \) what is the acceleration experienced by the ant?

\[
a_{\text{ant}} = \frac{v_{\text{ant}}^2}{R_{\text{ant}}} = \frac{4\pi^2 R_{\text{ant}}}{T_{\text{dvd}}^2} = \frac{4\pi^2 R_{\text{ant}}}{v^2 \cdot R_L^2}
\]

\[
= \frac{R_{\text{ant}}}{R_L^2} \cdot v^2 = \left[ a_{\text{ant}} = 229 \frac{m}{s} \right]
\]
Part 6: Aliens

The spaceship you command is about land on a planet. The computer finds the landing solution and when the spaceship is at a height $h$ ($t = 0s$) it sets the velocity of the spaceship to be $v_y(t) = -v_0y + kt^2$, where $h$, $v_0y$ and $k$ are all positive and known and the coordinate system assumes that the $\hat{y}$ axis goes from the surface of the planet up to the spaceship. All answers must be in terms of the known variables $h$, $v_0y$, and $k$.

(a) (2 points) Find the time it takes to land if the landing is supposed to be perfectly smooth (i.e. the vertical velocity must be zero at landing)

$$v_y(tg) = -v_0y + kt^2 = 0 \Rightarrow \quad t_g = \frac{\sqrt{v_0y}}{k}$$

(b) (4 points) Is the vertical acceleration in this problem constant or not? Is it positive or negative at the moment of landing?

$$a_y(tg) = \frac{d(-v_0y + kt^2)}{dt} = 2kt \Rightarrow \text{acceleration positive and not constant}$$

(c) (8 points) Find the position as a function of time and the height of the spaceship when half the landing time has elapsed.

$$y(t) = y_0 + \int_0^t v_y(t)dt = 2kt$$

$$\quad = h + \int_0^t (-v_0y + kt^2) dt$$

$$y(t) = h - v_0yt + \frac{k}{3}t^3$$

$\Rightarrow y(t_g/2) = h - v_0yt\frac{t_g}{2} + \frac{k}{24}(\frac{v_0y}{k})^{3/2}$

$$y(t_g/2) = h - \frac{v_0y}{2} \sqrt{\frac{v_0y}{k}} + \frac{k}{24} (\frac{v_0y}{k})^{3/2}$$