Fill out the information below but do not open the exam until instructed to do so!

Name:________________________
Signature:________________________
Student ID:________________________
E-mail:________________________
Section #:________________________

Rules of the exam:
1. You have the full class period to complete the exam.
2. Formulae are provided on the last page. You may NOT use any other formula sheet.
3. When calculating numerical values, be sure to keep track of units.
4. You may use this exam or come up front for scratch paper.
5. Be sure to put a box around your final answers and clearly indicate your work to your grader.
6. Clearly erase any unwanted marks. No credit will be given if we can’t figure out which answer you are choosing, or which answer you want us to consider.
7. Partial credit can be given only if your work is clearly explained and labeled.
8. All work must be shown to get credit for the answer marked. If the answer marked does not obviously follow from the shown work, even if the answer is correct, you will not get credit for the answer.

Put your initials here after reading the above instructions:__________

<table>
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<tr>
<th>Part</th>
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<td>Part 1 (15)</td>
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Part 1: Basic ideas of units, conversions, and vectors.

Problem 1.1: (1p) What system of units is used in this course? What are the basic units of mass, length, and time of that system?

International System (SI), Kilogram, meter, seconds.

Problem 1.2: Joule, erg and eV are units of energy defined as:

\begin{align*}
1 \text{ J (Joule)} &= 1 \text{ Kg m}^2/\text{s}^2 \\
1 \text{ erg (erg)} &= 1 \text{ gram cm}^2/\text{s}^2 \\
1 \text{ eV (electron-Volt)} &= 1.6 \times 10^{-12} \text{ erg}
\end{align*}

Question 1.2.1: (2p) Express 1 erg in units of Joules.

\begin{align*}
el_{\text{erg}} &= 1 \frac{g \text{cm}^2}{s^2} = 1 \frac{Kg \text{m}^2}{s^2} \left( \frac{m}{100} \right)^2 = \frac{1}{1000 \times 100 \times 100} \frac{Kgm^2}{s^2} = 10^{-7} \text{ J}
\end{align*}

Question 1.2.2: (2p) The LHC accelerator in Switzerland accelerates protons to the world’s largest energy of $3.5 \times 10^{12}$ eV. Express that energy in Joules.

\begin{align*}
3.5 \times 10^{12} \text{ eV} &= 3.5 \times 10^{12} \times 1.6 \times 10^{-12} \text{ erg} = 3.5 \times 1.6 \times 10^{-7} \text{ J} = 5.6 \times 10^{-7} \text{ J}
\end{align*}

Problem 1.3: The following plot shows the position $x$ as a function of time

Question 1.3.1: (5p) For each time range A,B,C...I, fill the table below writing in each cell whether the velocity and acceleration are <0, >0, or =0.

<table>
<thead>
<tr>
<th>Region</th>
<th>Velocity</th>
<th>Acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>=0</td>
<td>=0</td>
</tr>
<tr>
<td>B</td>
<td>&gt;0</td>
<td>&gt;0</td>
</tr>
<tr>
<td>C</td>
<td>&gt;0</td>
<td>=0</td>
</tr>
<tr>
<td>D</td>
<td>&gt;0</td>
<td>&lt;0</td>
</tr>
<tr>
<td>E</td>
<td>&lt;0</td>
<td>&lt;0</td>
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<tr>
<td>F</td>
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<td>=0</td>
</tr>
<tr>
<td>G</td>
<td>&lt;0</td>
<td>&gt;0</td>
</tr>
<tr>
<td>H</td>
<td>=0</td>
<td>=0</td>
</tr>
<tr>
<td>I</td>
<td>&gt;0</td>
<td>&gt;0</td>
</tr>
</tbody>
</table>

Question 1.3.2: (2p) Is the magnitude of the velocity greater in region C than it is in F? Why?

The magnitude of the velocity at a given time is the magnitude of the slope of the tangent line in the above graph at that given time. The slope at time range C is about +2 squares/2 squares, with a magnitude of +1. The slope at time range F is about -4 squares/2 squares with a magnitude of -2. Hence, the answer is NO; the magnitude of the velocity at region C is smaller than that at region F.
**Part 2:** A car departs from rest under a constant acceleration of 1 m/s² moving in a straight line. After travelling some distance it passes first a dancing club and 20 seconds later a gas station. The distance between the dancing club and the gas station is 400 meters.

**Question 2.1.1:** (6p) In the space below draw a schematic diagram of the problem and write any associated times. In addition choose and draw a coordinate system and clearly indicate its origin.

![Schematic Diagram](image)

**Question 2.1.2:** (4p) Write the equations of motion of the accelerating car according to your coordinate system.

\[ X_T(t) = \frac{1m}{2s^2} t^2 \]

**Question 2.1.3:** (6p) Find the time it took the car to travel from the original point of departure to the club. (Hint: use the fact that you know the distance and the time between the Club and the gas station)

\[
X_T(t_c + 20s) - X(t_c) = 400m \quad \Rightarrow \quad \frac{1m}{2s^2} (t_c + 20s)^2 - \frac{1m}{2s^2} t_c^2 = \frac{1m}{2s^2} 400s^2 + \frac{1m}{2s^2} 2t_c 20s = 400m
\]

\[
200m + \frac{20m}{s} t_c = 400m \quad \Rightarrow \quad t_c = \frac{200ms}{20m} = 10s
\]

**Question 2.1.4:** (4p) Find the distance between the dancing club and the original point from where the car departed.

\[ X_T(t_c) = \frac{1m}{2s^2} t_c^2 = \frac{1m}{2s^2} 100s^2 = 50m \]
Part 3: Acceleration in both components.

Problem 3.1: A car is fitted with a rocket propulsion engine that provides the car with a constant acceleration in the horizontal direction. As depicted below the car must jump of a 10m high bridge and land on a flatbed truck moving with a constant velocity of 20 m/s. At the moment the car leaves the bridge the truck is at a distance of 15m from the bridge and the car has an initial velocity of 20 m/s. Ignore the height of the flatbed, air resistance and any mass loss due to the rocket. The following questions must be answered in the form of a number with proper units.

Question 3.1.1: (3p) Choose and draw your coordinate system on the figure above and associate times to the different events.

Question 3.1.2: (5p) Write the position of the car and the truck as a function of time

For the truck:
\[ X_T(t) = 15m + 20 \frac{m}{s} t \]
\[ Y_T(t) = 0 \]

For the car:
\[ X_C(t) = 20 \frac{m}{s} t + \frac{a_x}{2} t^2 \]
\[ Y_C(t) = 10m - \frac{g}{2} t^2 \]

Question 3.1.3: (5p) Find the time at which the car lands on the truck.

\[ Y_C(t_L) = 10m - \frac{g}{2} t_L^2 = 0 \Rightarrow t_L = \sqrt{\frac{20m}{g}} = 1.43 \text{ s} \]

Question 3.1.4: (7p) Find the minimum horizontal acceleration that the rocket propulsion engine in the car needs to give the car so it can successfully land on the truck.

\[ X_C(t_L) = X_T(t_L) \text{ replacing we get} \]
\[ 20 \frac{m}{s} t_L + \frac{a_x}{2} t_L^2 = 15m + 20 \frac{m}{s} t_L \Rightarrow a_x = \frac{30m}{t_L^2} = g \frac{30m}{20m} = 14.7 \frac{m}{s^2} \]
Part 4: A more complex problem.

Problem 4.1: A ball is tied up to a rod of radius $R$ connected to a motor that makes it spin in the vertical plane with a uniform motion once every $T_A$ seconds. A second similar device is located at a distance $d$ and rotating in opposite direction with period $T_B$ as shown in the picture below. Gravity is present and the center of both devices is located a distance $h$ with respect to the ground. All answers must be expressed in terms of known parameters.

![Diagram of two balls colliding](image)

The vertical position where the particles collide is given by $Y_A(t_c)$ since I put my coordinate system at the center of the circle which is from where $h$ is measured it follows $h=-Y_A(t_c)$

$$h = -Y_A(t_c) = -R + \frac{g}{2} t_c^2 = -R + \frac{g}{2} \left( \frac{T_A T_B d}{2\pi R(T_B + T_A)} \right)^2$$

- **Part 4:**
  - **Problem 4.1:** A ball is tied up to a rod of radius $R$ connected to a motor that makes it spin in the vertical plane with a uniform motion once every $T_A$ seconds. A second similar device is located at a distance $d$ and rotating in opposite direction with period $T_B$ as shown in the picture below. Gravity is present and the center of both devices is located a distance $h$ with respect to the ground. All answers must be expressed in terms of known parameters.

  **Question 4.1.1:** (2p) Find the ratio of the speeds of the balls in their movement around their respective circles.

  $$\left| \frac{v_A}{v_B} \right| = \frac{2\pi R/T_A}{2\pi R/T_B} = \frac{T_B}{T_A}$$

  **Question 4.1.2:** (2p) Find the ratio of the magnitude of the acceleration of the balls in their movement around their respective circles. In general, what is the direction of the acceleration?

  $$\left| a_A \right| = \frac{v_A^2/R}{v_B^2/R} = \frac{T_B^2}{T_A^2}$$

  The acceleration vector of the balls point towards the center of their respective circles.

  **Question 4.1.3:** (7p) When both balls are simultaneously at their maximum heights the balls break free of their respective rods and start moving against each other. Find the time it takes the balls to collide assuming the height $h$ is big enough.

  $$X_A(t_c) = X_B(t_c) \Rightarrow v_A t_c = d - v_B t_c \Rightarrow t_c = \frac{d}{v_A + v_B} = \frac{d}{2\pi R \left( \frac{1}{T_A} + \frac{1}{T_B} \right)} \Rightarrow t_c = \frac{T_A T_B d}{2\pi R(T_B + T_A)}$$

  **Question 4.1.4:** (4p) In your coordinate system find the horizontal position at which both balls collide.

  $$X_A(t_c) = \frac{2\pi R}{T_A} t_c = \frac{2\pi R}{T_A} \frac{T_A T_B d}{2\pi R(T_B + T_A)} = \frac{T_B d}{(T_A + T_B)}$$

  **Question 4.1.5:** (5p) Find the minimum vertical distance $h$ necessary for the balls to collide in the air.

  The vertical position where the particles collide is given by $Y_A(t_c)$ since I put my coordinate system at the center of the circle which is from where $h$ is measured it follows $h=-Y_A(t_c)$

  $$h = -Y_A(t_c) = -R + \frac{g}{2} t_c^2 = -R + \frac{g}{2} \left( \frac{T_A T_B d}{2\pi R(T_B + T_A)} \right)^2$$
Formula sheet:

**Vectors:**

\[ \mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k} \]

\[ |\mathbf{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2} \]

\[ \tan(\varphi) = \frac{A_y}{A_x}, \text{ where } \varphi = \text{ angle between } \mathbf{x} \text{ axis and projection of vector } \mathbf{A} \text{ to the } (\mathbf{x} \mathbf{y}) \text{ plane.} \]

In 2-D, \( \varphi = \text{ angle between } \mathbf{x} \text{ axis and vector } \mathbf{A} \).

**Mathematical Formulae:**

\[ ax^2 + bx + c = 0 \implies t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

If \( x = at^n \) \( \implies \frac{dx}{dt} = nat^{n-1} \).

If \( x = at^n \) \( \implies \int_{t_1}^{t_2} x(t) \, dt = \frac{a}{n+1} \left( t_2^{n+1} - t_1^{n+1} \right) \)

**The following equations are always true:**

\[ \ddot{v} = \frac{d\ddot{r}}{dt} \implies v_x = \frac{dx}{dt} \]

\[ \dddot{a} = \frac{d\dddot{r}}{dt} \implies a_x = \frac{dv_x}{dt} \]

\[ \ddot{r}(t) = \ddot{x}_0 + \int_{0}^{t} \dddot{v}(t) \, dt \implies x(t) = x_0 + \int_{0}^{t} v_x(t) \, dt \]

\[ \dddot{v}(t) = \dddot{v}_0 + \int_{0}^{t} \dddot{a}(t) \, dt \implies v_x(t) = v_{x_0} + \int_{0}^{t} a_x(t) \, dt \]

\[ v_{\infty, x} = \frac{(x_2 - x_0)}{(t_2 - t_1)}, \quad a_{\infty, x} = \frac{(v_{x_2} - v_{x_0})}{(t_2 - t_1)} \]

**The following apply for constant acceleration:**

\[ \ddot{r} = \ddot{x}_0 + \dddot{v}_0 t + \frac{1}{2} \dddot{a} t^2 \implies x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2 \]

\[ \dddot{v} = \dddot{v}_0 + \dddot{a} t \implies v_x(t) = v_{x_0} + a_x t \]

\[ v_x^2(t) = v_{0x}^2 + 2a_x \Delta x \]

\[ \Delta x = \frac{v_{0x} + v_x(t)}{2} t \]

**Other Equations:**

\[ a_{rad} = \frac{v^2}{R} = \frac{4\pi^2R}{T^2} \]

\[ \ddot{v}_{PA} = \ddot{v}_{PB} + \ddot{v}_{BA} \]