Segway™

Two massless wheels of radius R are separated by a distance 2L and connected by a massless primary axis along the Y direction. The primary axis allows the wheels to move independently from each other. A massless secondary axis rigidly tees off from the middle point of the primary axis. The secondary axis lies in the X-Z plane an angle \( \alpha \) with respect to the Z axis. One end of the secondary axis contains a mass M a distance d to the junction of the axes, in the other end it contains a disk with moment of inertia I rotating with an angular velocity \( \omega \) along the secondary axis and towards the mass M. (Note: gravity the coordinate system is aligned to the system in the figure at \( t=0 \) to help visualize the system. If the system rotates then it will get de-aligned with the axis, as the coordinate axis must be inertial or non-accelerated.)

Solve the problem as if you want to win the Nobel price on clarity of exposition.

Answer all these questions:

1. Assuming for this question that the angle \( \alpha \) is zero, will the system start rotating or will it maintain its position?
2. For an angle \( \alpha \) different than zero, will the system start moving or will it maintain its position? If the system will start moving, what kind of movement is it going to do?
3. If your answer is that it will precess, then determine the axis of precession and the angular velocity of precession, also determine whether the precession is counter-clock wise or clock wise.
4. Finally, calculate the angular velocity of the wheels in terms of the precession velocity and the known parameters of the problem.
Solutions:

1. It will maintain its position.
2. It will start rotating around the z axis.
3. Precess around the z axis with an angular velocity $\Omega = \frac{Mgd}{I\omega}$. As looked from above (high Z looking down), counter-clock wise.
4. It should be close to $\omega = \Omega L/R$, verify it.