USEFUL INFORMATION

If \( f(x) = kx^n \quad \frac{df}{dx} = nkx^{n-1} \)

If \( f(x) = kx^n \quad \int_{A}^{B} f(x)dx = \frac{1}{n+1} k(B^{n+1} - A^{n+1}) \)

\[
\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{\text{tot}} \cdot d\vec{r} = \frac{1}{2} m v^2(\vec{r}_2) - \frac{1}{2} m v^2(\vec{r}_1)
\]

If \( \vec{F} \) is conservative:

\[
\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = -[U(\vec{r}_2) - U(\vec{r}_1)]
\]

and

\[
F_x = -\frac{\partial U}{\partial x} \quad F_y = -\frac{\partial U}{\partial y}
\]

\[
\vec{L} = \vec{r} \times \vec{p} \quad \vec{\tau} = \vec{r} \times \vec{F} \quad I = \sum m_i r_i^2
\]

1.

2.

3.

4.
1. (25 points) Derive the expressions for the $\vec{v}_r$ and $\vec{v}_\theta$ components of the velocity and acceleration.
2. (25 points) A mechanical device consists of a stationary "cam" and a rod rotating about a pivot point. The rod has a pin on it which slides along a slot. The pin is pushed against the cam by a spring.

The shape of the cam is given by \( r = r_0(1 + c_1 \cos \theta) \) where \( r_0 \) and \( c_1 \) are known constants. The rod rotates counterclockwise at a constant, known angular velocity \( \omega_0 \). Find the total force exerted on the pin, which has known mass \( m \). (Ignore gravity.)

Free Body Diagrams (If appropriate). Law or Definition

Application

Result
3. (25 points) A block of mass $M$ is suspended by two massless strings of length $S$. The block explodes into two pieces, one of mass $m_1 = \frac{M}{3}$ and the other of mass $m_2 = \frac{2M}{3}$ which both instantaneously go off horizontally. If the smaller piece swings up to a known, maximum angle $\theta_1$ what will be the maximum angle for the piece of mass $m_2$?

Free Body Diagrams (If appropriate). Law or Definition

Application

Result
4. (25 points) A block of mass $M$ is on a frictionless table that has a hole a distance $S$ from the block. The block is attached to a massless string that goes through the hole. A force $F$ is applied to the string and the block is given an angular velocity $\omega_0$, with the hole as the origin, so that it goes in a circle of radius $S$. The force $F$ is increased, starting at time $t = 0$, so that the distance between the block and the hole decreases according to $S - c_1 t^2$. Here $c_1$ is a known, positive constant. Assuming the string stays straight and can only pull along its length, find the torque about the hole exerted by each force acting on the block and the angular acceleration of the block as a function of time.

Free Body Diagrams (If appropriate). Law or Definition

Application

Result