USEFUL INFORMATION

If \( f(x) = kx^n \) then \( \frac{df}{dx} = nkx^{n-1} \)

If \( f(x) = kx^n \) then
\[
\int_A^B f(x) \, dx = \frac{1}{n+1} k(B^{n+1} - A^{n+1})
\]

\[
\int_{r_1}^{r_2} \vec{F}_{\text{tot}} \cdot d\vec{r} = \frac{1}{2} mv^2(r_2) - \frac{1}{2} mv^2(r_1)
\]

If \( \vec{F} \) is conservative:
\[
\int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} = -[U(r_2) - U(r_1)]
\]

and

\[
F_x = -\frac{\partial U}{\partial x} \quad F_y = -\frac{\partial U}{\partial y}
\]

\[
\vec{L} = \vec{r} \times \vec{p} \quad \vec{\tau} = \vec{r} \times \vec{F} \quad I = \sum m_i r_i^2
\]

DO NOT WASTE TIME DOING ARITHMETIC

1.

2.

3.

4.
1. (25 points) Derive the expressions for the $\vec{r}$ and $\vec{\theta}$ components of the velocity and acceleration.
2. (25 points) A man of mass $m_1$ sits on a sled, mass $m_2$ on the top of a frictionless hill of height $H$. The sled starts down the hill with an initial velocity $v_0$ directed towards the North.

![Diagram of a man sitting on a sled at the top of a hill with an arrow indicating the initial velocity towards the North, and another sled at the bottom with an arrow indicating the direction of magnitude $v_1$.]

a. What is the man's velocity at the bottom of the hill? Call it $\vec{v}_B$. 

b. At the bottom of the hill the surface is solid ice so that there is no friction. The man jumps off the sled onto another sled, mass $m_3$ which is at rest. The empty sled goes off at an angle $\theta_1$ with velocity of magnitude $v_1$. Obtain the necessary equations to determine the position of the man $T$ seconds after he jumps from one sled to the other? DO NOT SOLVE THE EQUATIONS!

![Diagram of the man jumping from one sled to another, with an arrow indicating the angle $\theta_1$ and the direction of magnitude $v_1$.]
3. (25 points) A massless rod can rotate without friction about a vertical axle. A small mass $m_1$ is fixed to the rod a distance $H$ from the axle. A second small mass $m_2$ is initially a distance $S$ from the first mass, as shown.

The rod and the masses are set into motion rotating about the axle with angular velocity $\omega_0$. At $t = 0$ $m_2$ begins to move towards $m_1$ so that the distance between them is $S - ct^2$ where $c$ is a known constant.

a. What will be the angular velocity of the rod as a function of time while $m_2$ is moving towards $m_1$?

b. What is the force that the rod exerts on $m_2$ while it is moving?

c. If the rod were not massless but instead had a moment of inertia $I_{rod}$ about the axle, what would be the angular velocity of the rod as a function of time while $m_2$ is moving towards $m_1$?
4. (25 points) An electron with mass $m$ and charge of magnitude $q_1$ is attracted to a proton, which is fixed at the origin, by a force of magnitude

$$F = \gamma \frac{q_1 q_2}{r^2}$$

where $\gamma$ is a known constant, $q_2$ is the charge of the proton and $r$ is the distance of the electron from the origin.

a. If the electron moves in the $x,y$ plane in a circle of radius $R$, what is its angular momentum about the origin?

b. If instead of moving in a circle the electron’s position was given by $r(t) = r(0) + c_1 t$, $\theta(t) = \theta(0) + c_2 t$ where $r(0)$, $c_1$, $\theta(0)$, and $c_2$ are known, what would be the kinetic energy of the electron?

c. Calculate the work done by the force exerted by the proton if the electron moves from the point $r = R, \theta = 0$ to the point $r = 2R, \theta = \frac{\pi}{4}$. 