USEFUL INFORMATION

If \( f(x) = kx^n \), then \( \frac{df}{dx} = nkx^{n-1} \)

If \( f(x) = kx^n \), then \( \int_A^B f(x)dx = \frac{1}{n+1}k(B^{n+1} - A^{n+1}) \)

If \( f(x) = kx^n \), then \( \int f(x)dx = \frac{1}{n+1}kx^{n+1} + C \)

\[
\int_{\vec{r}_1}^{\vec{r}_2} \vec{F}_{\text{tot}} \cdot d\vec{r} = \frac{1}{2}mv^2(\vec{r}_2) - \frac{1}{2}mv^2(\vec{r}_1)
\]

If \( \vec{F} \) is conservative:

\[
\int_{\vec{r}_1}^{\vec{r}_2} \vec{F} \cdot d\vec{r} = -[U(\vec{r}_2) - U(\vec{r}_1)]
\]

and

\[
F_x = -\frac{\partial U}{\partial x} \quad F_y = -\frac{\partial U}{\partial y}
\]

1. 
2. 
3. 
4. 


1. (25 points) Two blocks with masses, \( m_1 \) and \( m_2 \), are connected by a massless, unstretchable rope as shown below. There is a coefficient of friction \( \mu_1 \) between the surface and block 1 and a coefficient of friction \( \mu_2 \) between the surface and block 2. At \( t = 0 \) block 2 is started with an initial velocity of \( v_0 \) down the plane.

- Draw the free body diagram for each block.
- Find the tension in the string and the acceleration of each block.
2. (25 points) A small object of mass \( m \) is placed on the frictionless surface. There are two forces, \( \vec{F}_1 \) and \( \vec{F}_2 \), acting on the object that have components only in the \( x \) direction, given by

\[
F_{1x} = \frac{c_1}{x^2} \quad \text{and} \quad F_{2x} = c_2 \left( 1 - \frac{x}{S} \right)
\]

with \( c_1 \) and \( c_2 \) positive and \( S \) is the point shown. The object is placed at rest at the point \( x = S \).

How do \( c_1 \) and \( c_2 \) have to be related in order for the object to move to the right to the point \( x = 2S \) and then begin moving to the left?
3. (25 points) A spring is placed as shown at the bottom of a frictionless inclined plane which has the angle $\theta$. The spring does not follow Hooke's Law but, instead, the magnitude of the force exerted by the spring is $F_{Spring} = k_1 x + k_2 x^3$. Here $k_1$ and $k_2$ are known, positive constants and $x$ is the amount the spring is stretched or compressed. As usual a compressed spring pushes.

\[ \text{The length of the plane is } L \text{ and the uncompressed length of the spring is also } L. \text{ A projectile of mass } m \text{ is placed against the spring which is compressed by the amount } A. \text{ The projectile is then released from rest.} \]

a. Find the potential energy function for the force exerted by the spring.

b. Find the velocity of the projectile when it reaches the top of the inclined plane.

c. Find $H$, the maximum height the projectile will reach above the end of the plane.
4. (25 points) An object of mass \( m \) is placed at rest at the point \( x = 0 \) on a horizontal table and, at time \( t = 0 \), a horizontal force is applied to it given by \( F_x = c_1 t \) where \( c_1 \) is a known positive constant. The coefficient of friction between the table and the object is \( \mu \).

\[
\begin{array}{c}
\text{m} \\
X = 0 \quad X = A
\end{array}
\]

a. If the friction force is ignored what will be the object's velocity in the \( x \) direction when it reaches the point \( x = A \)?

b. If the friction force is not ignored find the object's velocity in the \( x \) direction as a function of time.