USEFUL INFORMATION

If \( f(x) = kx^n \), \( \frac{df}{dx} = nkx^{n-1} \)

If \( f(x) = kx^n \), \( \int f(x)dx = \frac{1}{n+1} kx^{n+1} + C \)

For the SPECIAL CASE:
CONSTANT ACCELERATION IN ONE DIMENSION

\[ x(t) = \frac{1}{2}a_c t^2 + v(0)t + x(0). \]

Please Note: The symbol \( g \) stands for the magnitude of the acceleration vector due to gravity and, as such, it is a positive quantity.

Do Not Spend Too Much Time on Algebra!

1.
2.
3.
4.
1. An object moves along the x-axis and is at the point $x = D$ at $t = 0$. The object is observed to have a velocity given by $v(t) = c_1 t + c_2 t^2$, where $c_1$ and $c_2$ are known constants.

   a. Find $x(t)$.

   b. Find $a(t)$.

   c. Express each of the three vectors shown below and their sum in terms of the unit vectors $\hat{i}$ and $\hat{j}$.

![Diagram of vectors](image_url)
2. You are going to shoot an arrow at an apple on top of your physics teacher's head. The apple is at a height $H$ above the ground a horizontal distance $D$ away from you. You fire the arrow from a height $S$.

a. If the arrow is fired with initial velocity $v_0$ at an angle $\theta$ with the horizontal, what is the position of the arrow as a function of time?

b. What does the relationship between $v_0$, $\theta$, $D$, $S$ and $H$ have to be in order to hit the apple? (No algebra please.)
3. A rocket is fired vertically upwards with a velocity of magnitude $v_R$ from the top of a building of height $H$. Due to the rocket's engine the rocket has an acceleration, including the effect of gravity, given by

$$|\vec{a}| = g - c_1 t,$$

so that at $t = 0$ the acceleration was vertically down.

Assume $c_1$ small enough so that the rocket reaches some maximum height and comes back down to the ground with $|\vec{a}|$ always positive.

a. Find the equation that could be solved for $T_{\text{top}}$, the time to reach the maximum height. DO NOT SOLVE!

b. Find the equation that could be solved for $T_{\text{grnd}}$, the time for the rocket to hit the ground. DO NOT SOLVE!
4. A car filled with bad guys is driving up a hill with a constant acceleration of magnitude \( a_1 \), starting at the bottom with velocity of magnitude \( v_1 \). It will enter a tunnel when it gets to the point \( H \) above the ground. A drone is to be launched from a tower the instant the car starts up the hill which is defined to be \( t = 0 \). The drone has initial velocity \( \vec{v}_m \) with magnitude \( v_m \) and the direction shown. The plan is for it to enter the tunnel the same time as the car, explode collapsing the tunnel and destroying all evidence of the drone’s existence. It is programmed to have a upwards vertical acceleration of magnitude \( c_1 t \) and a horizontal acceleration, to the left, of magnitude \( c_2 t^2 \). (Here \( c_1 \) and \( c_2 \) are positive constants. The effect of gravity is included in the given accelerations of the car and the drone.)

\[ Y \]

\[ \theta \]

\[ \vec{V}_m \]

\[ \vec{a} \]

\[ S \]

\[ \rightarrow x \]

a. Find the \( x \) and \( y \) components of the car’s position as a function of time.

b. Find the \( x \) and \( y \) components of the drone’s position as a function of time.

c. What is the condition that must be satisfied in order for this covert mission to be successful? (No algebra please.)