USEFUL INFORMATION

If \( f(x) = kx^n \), \( \frac{df}{dx} = nkx^{n-1} \)

\( \frac{df}{dx} = n \)

If \( f(x) = kx^n \), \( \int f(x)dx = \frac{1}{n+1}kx^{n+1} + C \)

For the SPECIAL CASE:
CONSTANT ACCELERATION IN ONE DIMENSION

\[ x(t) = \frac{1}{2}at^2 + v(0)t + x(0). \]

Do Not Spend Too Much Time on Algebra!
1a. (10 points) An object moves along the x-axis so that its position is given by

\[ x(t) = k_1 t^5 + k_2 t^2 + c_1 t + c_2. \]

Here \( k_1 \) and \( k_2 \) are known constants and \( c_1 \) and \( c_2 \) are unknown. The velocity of the object is measured at the known time \( t = T_A \) and found to have the value \( w \). The position is measured at \( t = 0 \) and found to have the value \( D \).

Determine \( c_1 \) and \( c_2 \) and the object’s acceleration.

1b. (15 points) A sign, having known mass \( M \) is to be attached to the ceiling and a wall by two thin wires as shown:

![Diagram of a sign with two wires at different angles]

Each wire has essentially zero mass and can not be stretched. One wire is horizontal and the other is at the angle \( \theta \). Find the tension in each of the wires. (The tension is the force that the wire exerts on the sign.)
2. (25 points) A rocket travels along a straight, vertical line. Starting at the time defined to be $t = 0$, at a point defined to be the origin, its vertical velocity is observed to be

$$v(t) = -k_1 t^2 + k_2 t$$

where $k_1$ and $k_2$ are known, positive constants. What will be the rocket's maximum height?
3. (25 points) A robot has been programmed so that it has an acceleration given by

\[ \ddot{a} = 0\hat{i} - \beta t \hat{j} \]

in the coordinate system below. Here \( \beta \) is a known constant. The robot is to be placed at the point \( x = 0, y = A \) at \( t = 0 \) and given some initial velocity, \( \vec{u}_1 \), so that it can pick up some object at the point \( x = C, y = D \). (Here \( A, C, \) and \( D \) are known.)

a. What is the robot's position as a function of time?

b. What must be the magnitude of the robot's initial velocity if it leaves the initial point at the known angle \( \theta \) as shown?
4. (25 points) Two boxes are at rest on a frictionless table. There is no friction between the two boxes. The box with mass $M_B$ will be crushed if a force is applied to it that has a magnitude greater than the known value $T_C$. A force of magnitude $P$ is to be applied to the first block, mass $M_A$, at the fixed, known angle $\theta$.

a. (20 points) What is the largest value that $P$ can have without the second box being crushed?

b. (5 points) Find the maximum value that $P$ can have without crushing the box if now the surface of the table is such that there is a coefficient of friction $\mu$ between the table and first box, $M_A$, but all other surfaces are still frictionless.