USEFUL INFORMATION

\[ \vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1q_2}{r^2} \hat{r} \]

\[ d\vec{B} = \frac{\mu_0 i}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^3} \]

\[ \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i}_x + \frac{dy}{dt}\hat{i}_y = \frac{dr}{dt}\hat{i}_r + r\frac{d\theta}{dt}\hat{i}_\theta \]

\[ \oint \vec{E} \cdot d\vec{r} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{S} \]

\[ C = \frac{Q}{V} = \frac{A\varepsilon_0}{d} \quad R = \rho \frac{l}{A} \]

\[ \int \vec{B} \cdot d\vec{S} = \pm Li \]

\[ \oint \vec{B} \cdot d\vec{r} = \mu_0 i_{\text{enclosed}} \]

1.

2.

3.

4.

Caution: You must define any variable, such as \( i \) and \( Q \), preferably on a figure. Failure to do so will result in the loss of a large amount of credit!
1. (25 points) A very long, straight wire lies along the $z$ axis. It has a circular cross section of radius $R$. It carries a current $i$ uniformly spread over the cross section flowing in the $+z$ direction. The center of the wire is located at the origin. Find the components of the magnetic field for all $x$ and $y$. 

\[ +z \bigcirc_{\text{(out)}} \]

Law

Application

Result
2. (25 points) A very thin wire lies in the $x,y$ plane. It has the shape shown below consisting of two circular segments, centered at the origin, connected by segments radially out from the origin. It carries a current $i$ as shown below. Find the magnetic field at the origin.

\[ \text{Law} \]

Application
3. (25 points) Two parallel, resistance free rails lie in the horizontal plane and are a distance $W$ apart. The are connected by a resistance free wire at one end. A rod, mass $m$ and length $L$ is placed at rest on the rails at the point defined to be $x = 0$. A magnetic field is turned on which points perpendicular to the rails in the direction shown. The strength of the magnetic field increases with time according to $B(t) = B_0 \alpha t$ where $\alpha$ and $B_0$ are known constants. What force would you have to apply, starting at $t = 0$, in order for the rod to remain at rest? (Ignore self inductance.)

\[ \left[ \begin{array}{c}
\text{Law} \\
X = 0 \\
\rightarrow +x \\
\end{array} \right] \quad \begin{array}{c}
\overrightarrow{H} \\
\end{array} \]

Application

Result
4. (25 points) There are two concentric, spherical conducting shells. The smaller one has inner radius \( A \) and the larger one has inner radius \( B \). Both have thickness \( T \). The inner one is given a charge \( Q_0 \). A very small resistor, \( R \), is connected from the inner to the outer with a switch that is closed at time \( t = 0 \). Find the charge on the inner shell as a function of time ignoring any self inductance. You should assume that because the resistor is so small, all charges will be distributed with spherical symmetry all the time.

Law

Application

Result