USEFUL INFORMATION

For two point particles

$$\vec{F} = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$$

$$\int_{\vec{r}_1}^{\vec{r}_2} \vec{E} \cdot d\vec{r} = - [V(\vec{r}_2) - V(\vec{r}_1)]$$

$$d\vec{r} = dx \hat{i}_x + dy \hat{i}_y = dr \hat{i}_r + r d\theta \hat{i}_\theta$$

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y}$$

DO NOT WASTE TIME ON COMPLICATED INTEGRALS

1.
2.
3.
4.
1. (25 points) A particle with known mass, m, and known, positive charge q, is placed at the point \((a, b)\). The force of gravity, magnitude mg, acts in the vertical, y direction. A second particle with unknown charge, \(q_1\), is fixed at the origin. What constant, horizontal electric field must be created so that the first charge will remain at rest?
2. (25 points) A charge $Q$ is uniformly spread along a quarter of circle of radius $R$. Find the electric force that this exerts on a particle of charge $q_1$ at origin, which is the center of the semi-circle.

If the charge were not uniformly spread but instead the charge per unit length was given by $\lambda(\theta)$, some function of $\theta$, how would the force be modified? (No words, just equations.)
3. (25 points) An electric field is measured in some region and found to have the form

\[ \vec{E} = \frac{C}{r^4} \hat{r} \]

where \( r \) is the length of the vector from the origin to a point located at \( \vec{r} \), \( \hat{r} \) is the unit vector along \( \vec{r} \) and \( C \) is a known constant.

a. Find the electric potential function for this electric field.

b. From this electric potential function find the \( x \) and \( y \) components of the electric field at any point \((x, y)\).
4. (25 points) Consider a surface which is a prism with one side in the x,y plane at $z = 0$.

a. If the electric field is given to be $\vec{E} = \alpha x^2 \hat{i}_x + \beta \hat{i}_y$ with $\alpha$ and $\beta$ constants, find the electric flux out of the triangular surfaces.

b. For the electric field given above find the electric flux out of the sloped rectangular surface.

c. If the electric field is given instead by

$$\vec{E} = \alpha z^2 \hat{i}_x + \beta \hat{i}_y$$

with $\alpha$ and $\beta$ constants, find the electric flux out of the triangular surface located at $x = W$. 