SO(10) model, Flavor Violation and the Recent D0 Result

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Outline

A Minimal SO(10) Model

Proton decay

Fermion masses, mixings and predictions

Lepton Flavor Violation

Gauge coupling unification

$B_s - \bar{B}_s$ and the recent D0 result

Conclusion
The Model...

The Yukawa superpotential involves the couplings of 16-dimensional matter spinors:

\[ W_Y = \frac{1}{2} h_{ij} \psi_i \psi_j H + \frac{1}{2} f_{ij} \psi_i \psi_j \Delta + \frac{1}{2} h'_{ij} \psi_i \psi_j D. \]

\( \psi_i \) (i denotes a generation index) with 10 (H), 126 (\( \Delta \)), and 120 (D) dim. Higgs fields:

- h and f are symmetric matrices and h' is an antisymmetric matrix due to the SO(10) symmetry.

The Higgs doublet fields not only exist in H, \( \Delta \), D, but also in other Higgs fields (e.g., 210) which are needed in the model.
The Model...

SO(10) gets broken by the following Higgs Fields:

210, 45, 54, 126, 126,

SU(3) \times SU(2) \times SU(2) \times U(1)

Flipped SU(5), SU(5) \times U(1)

SM (SU(3) \times SU(2) \times U(1))

Pati-Salam:  
SU(2)_L \times SU(2)_R \times SU(4)_C
The Model...

210 Higgs multiplet (\(\phi\)): is employed to break the SO(10) symmetry

\[ \Delta \text{126 Higgs multiplet (}\Delta) : \text{introduced as a vector-like pair and this field also contains a Higgs doublet.} \]

The VEV of this pair reduces the rank of SO(10) group, keep supersymmetry unbroken down to the weak scale.

Altogether, we have six pairs of Higgs doublets:

\[ \phi_d = (H_{10}^{d}, D_1^{d}, D_2^{d}, \Delta_d, \Delta_d, \phi_d), \]
\[ \phi_u = (H_{10}^{u}, D_1^{u}, D_2^{u}, \Delta_u, \Delta_u, \phi_u), \]

where superscripts 1, 2 of \(D_{u,d}\) stand for SU(4) singlet and adjoint pieces under the \(G_{422} = SU(4) \times SU(2) \times SU(2)\) decomposition.

The mass term of the Higgs doublets: \((\phi_d)_a(M_D)_{ab}(\phi_u)_b,\)

The lightest Higgs pair (MSSM doublets) has masses of the order of the weak scale.
The Model...

The Yukawa interaction includes mass terms of the quark and lepton fields as follows (using 10+120+126 Higgs fields):

\[ W_{\text{mass}}^{Y} = hH^{10}_{d}(qd^{c} + \ell e^{c}) + hH^{10}_{u}(qu^{c} + \ell \nu^{c}) + 1/\sqrt{3} f \Delta_{d}(qd^{c} - 3\ell e^{c}) + 1/\sqrt{3} f \Delta_{u}(qu^{c} - 3\ell \nu^{c}) + \sqrt{2} f\nu^{c}\nu c \Delta_{R} + \sqrt{2} f\ell\ell\Delta_{L} + h'D^{1}_{d}(qd^{c} + \ell e^{c}) + h'D^{1}_{u}(qu^{c} + \ell \nu^{c}) + 1/\sqrt{3} h'D^{2}_{d}(qd^{c} - 3\ell e^{c}) - 1/\sqrt{3} h'D^{2}_{u}(qu^{c} - 3\ell \nu^{c}), \]

where q, u\(^c\), d\(^c\), \(\ell\), e\(^c\), \(\nu c\) are the quark and lepton fields for the standard model, which are all unified into one spinor representation of SO\(\langle 10 \rangle\).

\[
Y_{u} = h + r_{2}f + r_{3}h' \\
Y_{d} = r_{1}(h + f + h') \\
Y_{e} = r_{1}(h - 3f + c_{e}h') \\
Y_{\nu} = h - 3r_{2}f + c_{\nu}h'
\]

\(r_{1,2,3} : \text{Higgs mixing}\)
The Model...

Imposing that the Lagrangian is invariant under a CP conjugation, the Yukawa couplings, \( h_{ij}, f_{ij} \) and \( h'_{ij} \) and all masses and couplings in the Higgs superpotential are all real.

The mixing of the lightest Higgs doublets with the Higgs doublets present in 120 involves a pure imaginary coefficient which will make the fermion masses hermitian in this model.

Very interesting property!

Solves the EDM problems of SUSY models.
Neutrino Mass

The VEVs of the fields
\( \Delta_R : (1, 1, 3) \) and \( \Delta_L : (1, 3, 1) \) give neutrino Majorana masses.

**Seesaw: EW scale^{2}/GUT scale ~ eV**

The light neutrino mass is obtained as
\[
    m_{\nu}^{\text{light}} = M_L - M_{\nu}^D \ M^{-1}_R \ (M_{\nu}^D)^T
\]

where \( M_{\nu}^D = Y_h <H_u> \), \( M_L = 2\sqrt{2}f <\Delta_L> \), \( M_R = 2\sqrt{2}f <\Delta_R> \).

Pure type II: \( M_L \) (In this talk)

Lazarides, Shafi, Wetterich,81, Mohapatra, Senjanovic,81
Proton Decay

- Generic prediction of most Grand Unified Theories
- Lifetime $> 10^{33}$ yr!
Nucleon decay

• Reach of partial lifetime
  – $p \rightarrow e^+ \pi^0$ up to $10^{35}$ yrs with Mton water Cherenkov (present SK limit: $5.4 \times 10^{33}$ yrs)
  – $p \rightarrow \nu K^+$ up to a few $10^{34}$ yrs with 100 kton liq. Ar and 50 kton liq. scintillator (present SK limit: $2.0 \times 10^{33}$ yrs)

• There is a lot of life in proton decay

• It is possible to suppress the decay rate, but in many cases proton decay is just around the corner: keep looking!

• Next step is significant!
Proton Decay

- The amplitudes mediated by GUT bosons (dimension 6 operators) become small

\[
\frac{qqql}{\Lambda^2} \quad \frac{d^c d^c u^c e^c}{\Lambda^2} \quad \frac{e^c u^c qq}{\Lambda^2} \quad \frac{\bar{d}^c \bar{u}^c q l}{\Lambda^2}
\]

\( \Lambda \) is the cutoff scale for the Standard Model: \( M_{\text{GUT}} \)

- processes mediated by the triplet higgsino emerge (dimension 5 operators)

\[
\text{Amp} = \lambda_u \lambda_d \frac{1}{M_{H^3}} \frac{1}{M_{\text{SUSY}}} \frac{\alpha_s}{2 \pi} \]

\( M_{H^3} : \text{Mass of colored Higgsino} \sim M_{\text{GUT}} \)
Proton decay Summary for SO(10)

Models of SO(10); Mohapatra et al, Raby et al, Pati and Babu, Senjanovic et al, Okada et al, Nath et al

SO(10) allows only small tan$\beta$ (< 3) and very large values of SUSY masses

Small tan$\beta$ is not preferred by Higgs mass and large values of SUSY masses also mean problem!

Situation is as bad as in SU(5)
Proton Decay in SO(10) Model

The proton decay is mediated by the colored Higgs triplets:
\[ \varphi_T + \varphi_T^\dagger : ((3, 1, -1/3) + \text{c.c.}) \text{ (CL operator)}; \]
\[ \varphi_C + \varphi_C^\dagger : ((3, 1, -4/3) + \text{c.c.}) \text{ (CR operator)}; \]

These Higgs triplets appear in:
\[ 10 + 120 + 126 + 126 + 210 \] multiplets.

We generate both LLLL (CL) and RRRR (CR) operators:
\[ -W_5 = C_{ijkl}^L q_k q_\ell q_i q_j + C_{ijkl}^R e^c_k u^c_\ell u^c_i d^c_j \]

These operators are obtained by integrating out the triplet Higgs fields,
\[ \varphi_T = (H_T, D_T, D'_T, \Delta_T, \Delta'_T, \phi_T) \] The fields with ‘‘ are decuplet, and the others are sextet or 15-plet under SU(4) decomposition.
\[ \varphi_C = (D_C, \Delta_C). \]
Proton Decay…

\[ W_{\text{trip}}^Y = h H_T^- (q\bar{\ell} + u^c d^c) + h H_T (1/2qq + e^c u^c) + f \Delta_T^- (q\bar{\ell} - u^c d^c) + \ldots \]

Same \( h, f \) and \( h' \) which appear in the Yukawa couplings

\[ C_{ijkl}^L = c h_{ij} h_{kl} + x_1 f_{ij} f_{kl} + x_2 h_{ij} f_{kl} + x_3 f_{ij} h_{kl} + x_4 h'_{ij} h_{kl} + x_5 h'_{ij} f_{kl}, \]
\[ C_{ijkl}^R = c h_{ij} h_{kl} + y_1 f_{ij} f_{kl} + y_2 h_{ij} f_{kl} + y_3 f_{ij} h_{kl} + y_4 h'_{ij} h_{kl} + \ldots \]

\( c = (M_T^{-1})_{11} \), and the other coefficients \( x_i, y_i \) are also given by the components of \( M_T^{-1} \).

The proton decay amplitude:
\[ A = \alpha_2 \beta_p / (4\pi M_T m_{\text{SUSY}}) A_x, \text{ where} \]
\[ A_x = c A_{hh} + x_1 A_{ff} + x_2 A_{hf} + x_3 A_{fh} + x_4 A_{h'h} + x_5 A_{h'f} + \ldots \]
One way to suppress the decay amplitude is by demanding cancellation among different terms.

In order to achieve that, we need a cancellation among $h, f$ and $h'$ to have small couplings.

However, we also need cancellation among the same set of couplings to generate the large mass hierarchy among the quark masses, i.e.,

\[
Y_u = h + r_2 f + r_3 h'
\]
\[
Y_d = r_1 (h + f + h')
\]
\[
Y_e = r_1 (h - 3 f + c_e h')
\]
\[
Y_\nu = h - 3 r_2 f + c_\nu h'
\]
Understanding the solutions:

h, f, h’ combine together to produce the fermion masses

We need small numbers in this combination to reproduce the first generation fermion masses

Two ways: (1) Big h element+ Big f element + Big h’ element = Small element
(2) small h element+ small f element + small h’ element = Small element

Small elements are preferred to solve proton decay problem!

If the couplings are small then the amplitudes are small
Proton Decay...

To suppress $A_{hh}$, the elements $h_{11}$ and $h_{22}$ (in h-diagonal basis) are needed to be suppressed rather than the up- and charm-quark Yukawa couplings, respectively. As a result, we need Yukawa texture to be

$$h \simeq \text{diag}(-0,-0,O(1)).$$

Once $h$ is fixed, the other matrices $f$ and $h'$ are almost determined as

$$\overline{f} = \begin{pmatrix}
\sim 0 & \sim 0 & \lambda^3 \\
\sim 0 & \lambda^2 & \lambda^2 \\
\lambda^3 & \lambda^2 & \lambda^2
\end{pmatrix}, \quad \overline{h'} = -i \begin{pmatrix}
0 & \lambda^3 & \lambda^3 \\
-\lambda^3 & 0 & \lambda^2 \\
-\lambda^3 & -\lambda^2 & 0
\end{pmatrix}$$

where $\lambda \sim 0.2$.  

$Y_u = h + r_2 f + r_3 h'$

$Y_d = r_1 (h + f + h')$

$Y_e = r_1 (h - 3f + c_e h')$

$Y_\nu = h - 3r_2 f + c_\nu h'$
Proton Decay…

One example for numerical fit for 
$\tan \beta(M_Z) = 50, \tilde{h} = \text{diag}(0, 0, 0.638)$,

$$\tilde{f} = \begin{pmatrix}
\sim 0 & 0.0044 & 0.00208 \\
0.0044 & 0.00945 & 0.0101 \\
0.00208 & 0.0101 & 0.0071
\end{pmatrix} \quad \tilde{h}' = i \begin{pmatrix}
0 & -0.0022 & 0.00046 \\
0.0022 & 0 & 0.0181 \\
-0.00046 & -0.0181 & 0
\end{pmatrix}$$

$r_1 = 0.966, r_2 = 0.135, r_3 = 0, |c_e| = 0.987$. $r_2 \neq 0$ to produce correct charm mass

[ SU(5) like vacuum]

The coefficients, $x_i, y_i$, involve the colored Higgs mixings, which can be suppressed by our choice of the vacuum expectation values and the Higgs couplings.

The $A_{hh}$ for $p \rightarrow K\nu_\mu$ mode is $\sim 2 \cdot 10^{-11}$.

The Model Predictions

The number of parameters in the models is 17  
3 (h), 6(f), 3(h') and 5 Higgs parameters (r_{1,2,3}, c_e,c_{\nu}).

Explanation of the proton decay fix some parameters.  
We choose \( h_{11,22} = 0 \) and \( r_3 = 0 \).

Since we will be working pure type II seesaw, i.e., \( M_\nu = f v_L \), 
\( c_\nu \) is redundant in fitting fermion masses and mixings. This reduces 
the number of parameters to 13.

13 input parameters: up-type quark masses, charged lepton 
masses, the CKM angles and the phase, the ratio of the squared 
of neutrino mass differences (\( m^2_{\text{sol}}/m^2_{\text{A}} \)), and the bi-maximal 
mixings as input parameters.  
The down-type quark masses, U_{e3} and \( \delta_{\text{MNSP}} \) etc are the 
predictions of this model.
Strange Quark Mass

The predicted value of strange quark mass has two separate regions, roughly $m_s \sim \frac{1}{3}m_\mu (1 \pm O(\lambda^2))$. The negative sign corresponds to a strange quark mass:

$$m_s(\mu = 2\text{GeV}) = 120 - 130 \text{ MeV}.$$  

The lattice derived value, $m_s(\mu = 2\text{GeV}) = (105 \pm 25) \text{ MeV}$

The positive signature gives the following value of the strange quark mass, $m_s(\mu = 2\text{GeV}) = 155 - 165 \text{ MeV}$

$$\frac{m_s}{m_d} = 17-18, 19-20.5$$

[18.9 ± 0.8, Leutwyler’00]
\[ |U_{e3}| \]

We get the following approximate relation for \( U_{e3} \):
\[ |U_{e3}|^2 \approx \frac{\tan^2 \theta_{\text{sol}}}{1 - \tan^4 \theta_{\text{sol}}} \frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\Delta}} \]

We also have the following relation since \( U_{e3} \) is related to the ratio:
\[ |U_{e3}| \approx 1/\sqrt{2} |V_{ub}/V_{cb}| \]
The MNSP phase is given by the approximate expression:

\[
\sin \delta_{\text{MNSP}} \sim \frac{1}{\sqrt{2}} \sin \theta^e_{12} \sin \theta^\nu_{13} \sin (\tan^{-1} c e h'_{12} / (3 f_{12}))
\]

Gauge Coupling Unification

The minimal SO(10) model is ruled out when gauge coupling unification is required.


Fermion mass hierarchies plus the SO(10) breaking vacuum causes problem to gauge coupling unification

The minimal SO(10) superpotential is used, i.e., 210, 10 and 126 [K. Babu, R. Mohapatra, Phys.Rev.Lett.70:2845, 1993]

Proton decay also applies strong constraint

Soln.: Extension is needed, extra 120, 10 etc.
Gauge Coupling Unification…

We can solve this puzzle using 120 or even with an additional 10’ Higgs extension

The SO(10) symmetry breaks at around $10^{17-18}$ GeV

We also need the colored Higgs masses to be at the string scale for successful unification

This automatically solves proton decay problem.
Gauge Coupling Unification...

The strategy is to find the lighter multiplets to keep the gauge coupling unification at the string scale.

The lighter multiplets which can move the gauge coupling unification to $10^{17-18}$ GeV are the followings:

$$(8,1,0); (8,2,1/2)+cc; (6,1,1/3)+cc; (6,2,-1/6)+cc \ [210,126+126]$$

The colored Higgs masses are of the string scale $\Rightarrow$ proton decay is suppressed.

The new multiplets can be around $10^{15-16}$ GeV and the unification scale is the string scale $\Rightarrow$ Flavor violation due to the majorana couplings [when the fields from $126+126$ are light].

Gauge Coupling Unification and Proton Decay

Proton decay amplitudes are reduced, since the Cutoff scale are high

\[ \lambda_u \lambda_d \frac{1}{M_{H^c}} \frac{1}{M_{SUSY}} \alpha_s/2 \pi \]

\( M_{H^3} \) becomes large

The unification of the three gauge couplings provides two independent relations on the particle mass spectrum below the symmetry breaking scale

\[
-2\alpha_3^{-1}(m_Z) + 3\alpha_2^{-1}(m_Z) - \alpha_1^{-1}(m_Z) \\
= \frac{1}{2\pi} \left( \frac{12}{5} \ln \frac{M_{H^c}}{m_Z} + \sum_I N^I_A \ln \frac{M_I}{\Lambda} - 2 \ln \frac{m_{SUSY}}{m_Z} \right),
\]

\[
-2\alpha_3^{-1}(m_Z) - 3\alpha_2^{-1}(m_Z) + 5\alpha_1^{-1}(m_Z) \\
= \frac{1}{2\pi} \left( 12 \ln \frac{M^2_{X}}{m_Z^2} + \sum_I N^I_B \ln \frac{M_I}{\Lambda} + 8 \ln \frac{m_{SUSY}}{m_Z} \right),
\]

\( N^I_A = 2T^3(\phi^I) - 3T^2(\phi^I) + T^1(\phi^I) \) and \( N^I_B = 2T^3(\phi^I) + 3T^2(\phi^I) - 5T^1(\phi^I) \)

Hisano, Murayama and Yanagida'93
Proton Decay…

Case 1: no $\phi$’s; $M_{Hc} \sim 10^{15}$ GeV: bad for proton decay

SU(5) got ruled out…

Case 2: with $\phi$’s; $M_{Hc}$ can be large (using positive $N^I$): good for proton decay

We rescue SO(10)…
How general are these solutions?

Can we rescue any SO(10)?

Yes, we can! Provided we have the following multiplets around $10^{16}$ GeV

Only these four fields can rescue SO(10)

<table>
<thead>
<tr>
<th>$(8, 2, 1/2) + c.c.$</th>
<th>$N_A$</th>
<th>$N_B$</th>
<th>SO(10)</th>
<th>SU(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(6, 1, 1/3) + c.c.$</td>
<td>$\frac{24}{5}$</td>
<td>24</td>
<td>126 + $\overline{126}$, 120</td>
<td>45, 50</td>
</tr>
<tr>
<td>$(6, 2, -1/6) + c.c.$</td>
<td>$\frac{12}{5}$</td>
<td>36</td>
<td>210</td>
<td>40</td>
</tr>
<tr>
<td>$(8, 1, 0)$</td>
<td>6</td>
<td>6</td>
<td>210, 45, 54</td>
<td>24, 75</td>
</tr>
</tbody>
</table>
Gauge Coupling Unification…

The Unification

$\alpha_3$, $\alpha_2$, and $\alpha_Y$ are around $10^{16}$ GeV
Since the colored Higgs fields can be heavier than $10^{17}$ GeV and the current nucleon decay bounds can be satisfied.

If $\tan \beta$ is large enough $>20$, the proton decay via dimension five operator (such as $p \rightarrow K^-\nu$) can be observable in the megaton class detector.

However, the proton decay via the dimension six operator (such as $p \rightarrow \pi e$) may not be observed.
Flavor violations:

If $(8, 2, 1/2)$ and/or $(6, 1, 1/3)$ is much lighter than the SO(10) breaking scale, a sizable flavor violation can be generated since those fields originate from 126 or 120 which couple to fermions.

The couplings can be written as:

\[ f q^c u(8,2,1/2) + f q^c d(8,2,-1/2) + f q q(6,1,-1/3) + f u^c d(6,1,1/3) \]

If $(8, 2, 1/2)$ field is light, it can generate off-diagonal elements for both left- and right handed squark mass matrices \([f_{23} \sim f_{33}]\).

If the light $(6, 1, -1/3)$ field comes form 126, it can generate off-diagonal elements only for left-handed squarks.

If both left- and right-handed squark mass matrices have sizable off-diagonal elements, the meson mixing via box diagram is enhanced and thus, it can have impact on $D^-\bar{D}$, $B_s^-\bar{B}_s$ mixings etc.
However, $f_{16 \cdot 16} H_{126}$ coupling has a source of large mixings.

The coupling includes the Majorana couplings: $f_L L L \Delta_L + f_R L^c L^c \Delta_R$

$$m_{\tilde{Q}}^2 \simeq m_{\tilde{Q}}^2 \simeq m_{\tilde{U}}^2 \simeq m_{\tilde{D}}^2$$

$$m_{\tilde{Q}}^2 \simeq m_0^2 \left(1 - \kappa U \begin{pmatrix} k_1 \\ k_2 \\ 1 \end{pmatrix} U^\dagger\right)$$

Threshold parameter: $\kappa \simeq \frac{(f_{33}^{\text{diag}})^2}{8\pi^2} \left(3 + \frac{A_0^2}{m_0^2}\right) \ln \frac{M_*}{M_{\text{GUT}}}$

$$f = U f^{\text{diag}} U^T$$

$M_*$: String/Planck scale

$$m_{\tilde{D}}^2_{23} = -\frac{1}{2} m_0^2 \kappa \sin 2\theta_{23} e^{i\alpha}$$

$$k_2 \simeq \frac{\Delta m^2_{\text{sol}}}{\Delta m^2_{\text{atm}}}$$

Both left- and right-squarks have sizable FCNC effects!
SUSY contributions in $B$-$\bar{B}$ mixings

\[ M_{12} = \langle B|H|\bar{B} \rangle \quad \Delta M = 2 |M_{12}| \]

The gluino box diagram dominates.

Mass insertion approximation:

\[
\frac{M_{12}^{\text{SUSY}}}{M_{12}^{\text{SM}}} \approx a\left[(\delta_{LL}^d)^2_{32} + (\delta_{RR}^d)^2_{32}\right] - b(\delta_{LL}^d)_{32}(\delta_{RR}^d)_{32} + \cdots
\]

\[ a \sim O(1), \ b \sim O(100) \text{ for } m_{\text{SUSY}} \sim 1 \text{ TeV} \]

(Randall-Su; Ball-Khalil-Kou)

\[
\delta_{LL,RR}^d = (M_d^2)^{LL,RR}/\bar{m}^2 \quad \bar{m} : \text{average squark mass}
\]

\[
(\bar{d}_L, \bar{d}_R) \begin{pmatrix} (M_d^2)^{LL} & (M_d^2)^{LR} \\ (M_d^2)^{RL} & (M_d^2)^{RR} \end{pmatrix} \begin{pmatrix} \bar{d}_L^\dagger \\ \bar{d}_R^\dagger \end{pmatrix}
\]

\[
(M_d^2)^{LL} = m_Q^2 + \cdots \quad (M_d^2)^{RR} = (m_{Dc}^2)^T + \cdots
\]
Large \( \tan\beta \)

Double penguin diagram can have large Contribution

\[ \sim \kappa^2 \tan^4 \beta / M_A^4 \]

This diagram’s contribution \( \sim \kappa^2 \tan^4 \beta / M_A^4 \)

Both SU(5) and SO(10) get contribution from this diagram

Chargino and gluino mediated penguins contribute

Similar single penguin diagrams contribute to \( \text{Br}[B_s \to \mu\mu] \)

\( \sim \kappa^2 \tan^6 \beta / M_A^4 \)

Large \( B_s \) phase induces larger \( B_s \to \mu\mu \) process even for \( \tan\beta \sim 20 \) for universal boundary condition

[Hamzaoui, Pospelov and Toharia; Buras, Chankowski, Rosiek and Slawianowska]
New Physics Contribution in the Mixing

Accurate measurement of mass difference is consistent with SM. \[ \Delta M_s = 17.77 \pm 0.10 \pm 0.07 \text{ ps}^{-1} \] [Tevatron]

One may think that large SUSY contribution is not allowed.

However, experimental result of \( \Delta M_s = 2|M_{12}(B_s)| \) does not constrain size of SUSY contribution \( |M_{12}^\text{SUSY}| \) much.

\[
M_{12} = M_{12}^{\text{SM}} + M_{12}^{\text{SUSY}}
\]

\[\arg M_{12}^{\text{SUSY}} \text{ is free in the model}\]

Due to free phases in the Yukawa coupling

There is room for a sizable SUSY contribution.

What is this large SUSY contribution?
When \( A_s^{SM} \approx A_s^{SM+NP} \),

\[
\sin \phi_{B_s} \approx \frac{1}{2} \frac{A_s^{NP}}{A_s^{SM}}
\]

Experimental bound for \( \Delta M_s \)

\( (f_{B_s}^2 B_{B_s} \) ambiguity from lattice)

\[
\frac{M^{full}_{12}}{M^{SM}_{12}} = \frac{A_s^{SM} e^{-2i\beta_s} + A_s^{NP} e^{2i(\phi_s^{NP} - \beta_s)}}{A_s^{SM} e^{-2i\beta_s}} \equiv C_{B_s} e^{2i\phi_{B_s}}
\]
SU(5) GUT

Down quarks \((D^c)\) and lepton doublet \((L)\) are unified in \(\bar{5}\).

\[
Q, U^c, E^c : 10 \quad \text{Right-handed neutrino} : N^c
\]

\[
W_Y = Y_u 10 \cdot 10 H_5 + Y_d 10 \cdot \bar{5} H_{\bar{5}} + Y_\nu \bar{5} N^c H_5
\]

Both RH down-squarks and LH sleptons can have FCNC effects.

(Moroi, Akama-Kiyo-Komine-Moroi, Baek-Goto-Okada-Okumura, …)
SU(5) GUT...

\[ m_5^2 \approx m_0^2 \left( 1 - \kappa U \begin{pmatrix} k_1 & \alpha Y_{\nu} Y_{\nu}^\dagger \alpha^{-1} \\ k_2 & 1 \end{pmatrix} \right) U^\dagger \]

\[ Y_e = Y_e^{\text{diag}} \]
\[ Y_\nu = U Y_\nu^{\text{diag}} U_R^\dagger \]
\[ M_N = M_N^{\text{diag}} \]

\[ m_5 = m_{\bar{D}c} = m_{\bar{L}} \]
\[ m_{10} = m_{\bar{Q}} = m_{\bar{U}c} = m_{\bar{E}c} \]
\[ U : \text{unitary mixing matrix} \]
\[ k_1, k_2 \ll 1 \]

\[ \kappa \approx \frac{(Y_{\nu}^{\text{diag}})_{33}^2}{8\pi^2} \left( 3 + \frac{A_0^2}{m_0^2} \right) \ln \frac{M_*}{M_{\text{GUT}}} \]
\[ (m_5^2)_{23} = -\frac{1}{2} m_0^2 \kappa \sin 2\theta_{23} e^{i\alpha} \]

\[ U \text{ is the MNS neutrino mixing matrix.} \]

The flavor violating elements in \( m_5^2 \) induces \( B_s \) mixing.
SU(5) GUT...

Large $m_5, m_{10}, \mu$ are needed to suppress $\tau \rightarrow \mu \gamma$.

Sparticle spectrum is restricted.  LHC
Both left- and right-squarks have FCNC effects in SO(10).

\[ \frac{M_{12}^{\text{SUSY}}}{M_{12}^{\text{SM}}} \approx a\left[ (\delta_{LL}^d)^2_{32} + (\delta_{RR}^d)^2_{32} \right] - b(\delta_{LL}^d)_{32}(\delta_{RR}^d)_{32} + \cdots \]

\[ a \sim O(1), \quad b \sim O(100) \quad \text{for} \quad m_{\text{SUSY}} \sim 1 \text{ TeV} \]

Flavor violating effects are larger in the box diagram in SO(10).

Only \( \delta_{RR}^d \) is large in SU(5).

Large Phase of $B_s$-$\bar{B}_s$ mixing

CP violation in $B_s \to J/\psi \phi$ decay ($b \to s c \bar{c}$).

$$S_{b \to sc\bar{c}} = \sin \phi_s$$

SM prediction: $\phi_s = 2\beta_s \simeq 0.04 \text{ (rad)}$ small!

$$\beta_s \equiv \arg \left( -\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*} \right)$$

Measurements:

$$\phi_s^{(CDF)} = \pm 0.28 - 1.29 \text{ (2.8 fb}^{-1}) \quad \text{arXiv:0810.3229}$$

$$\phi_s^{(D0)} = 0.57^{+0.30}_{-0.24} \text{ (stat)}^{+0.02}_{-0.07} \text{ (syst)} \quad (2.8 \text{ fb}^{-1})$$

2.2 sigma deviation from the SM \hspace{0.5cm} \text{(arXiv: 0802.2255)}

Combined data analysis by UTfit: More than 3 sigma deviation from the SM

$$\phi_{B_s} = -19.9 \pm 5.6 \text{ (degree)} \quad \text{arXiv:0803.0659}$$
Large Phase of $B_s - \bar{B}_s$ mixing

$\Phi_s = \frac{A_{NP}}{A_{SM}}$

$m_5 = m_{10}$ (GeV)

$\mathrm{Br}(\tau \to \mu \gamma) < 4.5 \times 10^{-8}$
$\tan \beta = 10$
$m_{1/2} = 300$ GeV
$\mu = 500$ GeV

Dutta and Mimura,
Large Phase of $B_s$-$\bar{B}_s$ mixing : Semileptonic $B \rightarrow X+\mu$

New Fermilab results from D0

\[ A_{sl}^b = \frac{N_{b}^{++} - N_{b}^{--}}{N_{b}^{++} + N_{b}^{--}} \]

$N_{b}^{++}$, $N_{b}^{--}$: number of events containing two $b$ hadrons decaying semileptonically

\[ A_{sl}^b : \] equivalent to the charge asymmetry of semileptonic decays of $b$ hadrons to wrong charge muons that are induced by oscillation

\[ A_{SL}^b = \frac{\Gamma ( \bar{B} \rightarrow \mu^+ + X ) - \Gamma ( B \rightarrow \mu^- + X )}{\Gamma ( \bar{B} \rightarrow \mu^+ + X ) + \Gamma ( B \rightarrow \mu^- + X )} = a_{sl}^b \]

\[ A_{sl}^b = (0.506 \pm 0.043)a_{sl}^d + (0.494 \pm 0.043)a_{sl}^s \]

\[ a_{sl}^d (SM) = (-4.8^{+1.0}_{-1.2}) \times 10^{-4} \]

\[ a_{sl}^s (SM) = (2.1 \pm 0.6) \times 10^{-5} \]

\[ A_{sl}^b (SM) = (-2.3^{+0.5}_{-0.6}) \times 10^{-4} \]

\[ A_{sl}^b = -0.00957(stat) \pm 0.00251 \pm 0.00146(syst) : Exptal : 3.2 \sigma \]
New Fermilab results from D0...
New Fermilab results from D0...

Using the experimental range for $a^d_{SL}$
Effect of Large Phase: e.g., $\phi_{BS} \sim 0.5$

$\phi_s \sim 0.5$

**Tension???

In order to obtain the fit, experimental Value is used  \[ a_{SL}^d : (-4.6 \pm 4.7) \times 10^{-3} \]

If we use the theoretical SM value (-4.8x 10^{-4}), within 1 \( \sigma \) of \( A_{SL}^b \)

\( a_{SL}^s \) has to be at least -0.012 or smaller

\[ \rightarrow \sin \phi = -2.5 \pm 1.3 \]

More than 1 \( \sigma \) away from \(|\sin \phi| < 1\)

1. **New contribution to** \( a_{SL}^d \): But Sin 2\( \beta \) is well measured

2. \[ a_{SL}^s = 2 \frac{|\Gamma_{s}^{12}|}{|\Delta M_{s}^{SM}|} \frac{\sin \phi}{\Delta} \]  
   *Where* \( \Delta = 0.97 \pm 0.26; \Gamma_{s}^{12} (SM) = 1/2(0.096 \pm 0.04) ps^{-1} \)

One can consider new physics contribution in \( \Gamma \): need to compete with CKM favored tree level contribution to \( b \rightarrow c\bar{c}s \)
Conclusion

We should wait for CDF to confirm this semileptonic asymmetry

The existence of deviation in the CP asymmetry measurement of $B \to J/\psi \phi$ from both CDF and D0 has made a good case for new physics

SO(10) model has a good chance to explain this

We showed an example of an SO(10) model which does not have proton decay problem, explains the masses and mixings of fermions