1. The Maxwell equations in the Lorentz covariant formulation of classical electrodynamics are:

\[ \partial_\mu F^{\mu\nu} = \frac{1}{c} j^\nu \] and \[ \partial_\mu \ast F^{\mu\nu} = 0 \] (1)

where \( F^{\mu\nu} \) is the electromagnetic field tensor and \( j^\mu \) is the charge-current density four vector and \( \ast F^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \) is the dual field strength. Show that the second equation in Eq. 1 can be written as \( \partial_\rho F^{\rho\mu} + \partial_\nu F_{\mu\nu} + \partial_\mu F_{\nu\rho} = 0 \).

2. Consider a system of particles labeled \( n \) with energy momentum four vectors \( p_n^\alpha(t) \). The energy momentum tensor can be written for such a system as:

\[ T^{\alpha\beta} = \sum_n p_n^\alpha(t) \frac{dx_n^\beta}{dt} \delta^3(x - x_n(t)) \]

where \( T^{00} = \sum_n c p_n^0(t) \delta^3(x - x_n(t)) \) and \( T^{0i} = \sum_n p_n^0(t) \frac{dx_n^i}{dt} \delta^3(x - x_n(t)) \).

Show that (i) \( \frac{\partial}{\partial x^\beta} T^{\alpha\beta} = 0 \) when the particles are free (i.e., without any force)

(ii) \( \frac{\partial}{\partial x^\beta} T^{\alpha\beta} = G^\alpha \) when there is force, where \( G^\alpha = \sum_n \delta^3(x - x_n(t)) \frac{dp_n^\alpha(t)}{dt} = \sum_n \delta^3(x - x_n(t)) \frac{dx_n^\alpha(t)}{dt} f_n^\alpha(t) \)

(iii) Since for electromagnetic force \( f^\alpha = e c F^{\alpha\gamma} dx_\gamma^\gamma \), show that, for e.m. \( \frac{\partial}{\partial x^\beta} T^{\alpha\beta} = \frac{1}{c} F^{\alpha\gamma} J_\gamma \).

3. In the class we have mentioned \( T^{\alpha\beta}_{em} = F^{\alpha\gamma} F_{\beta\gamma} - \frac{1}{4} \eta^{\alpha\beta} F_{\gamma\delta} F^{\gamma\delta} \). Now if \( T^{\alpha\beta} = \sum_n p_n^\alpha(t) \frac{dx_n^\beta}{dt} \delta^3(x - x_n(t)) + T^{\alpha\beta}_{em} \) show that \( \frac{\partial}{\partial x^\gamma} T^{\alpha\beta} = -\frac{1}{c} F^{\alpha\gamma} J_\gamma \) (using Eq. 1), but \( \frac{\partial}{\partial x^\beta} T^{\alpha\beta} = 0 \) i.e., net energy-momentum tensor is conserved.