1. Show
(a)\(V^\mu U_\mu\) transforms as a scalar if \(V^\mu\) is a contravariant vector and \(U_\mu\) is a covariant vector
(b)\(V^\mu U_\lambda\) transforms as a second order tensor with one contravariant and one covariant indices
(c) How does \(u_\mu V^\mu\) transform if \(V^\mu\) and \(U_\mu\) are both covariant vectors.

2. Given the numbers
\[A^0 = 5, \quad A^1 = 0, \quad A^2 = 1, \quad A^3 = 6, \quad B^0 = 0, \quad B^1 = 2, \quad B^2 = 4, \quad B^3 = 0,\]
\[C_{00} = 1, \quad C_{01} = 0, \quad C_{02} = 2, \quad C_{03} = 3, \quad C_{10} = 1, \quad C_{11} = 2, \quad C_{12} = 2, \quad C_{13} = 0, \quad C_{21} = 5, \quad C_{22} = 2, \quad C_{23} = 2, \quad C_{30} = 4, \quad C_{31} = 1, \quad C_{32} = 3, \quad C_{33} = 0,\]
find: (a) \(A^\alpha B_\alpha\); (b) \(A^\alpha C_{\alpha\beta}\) for all \(\beta\)

3. The following matrix gives a Lorentz transformation \(\Lambda^\mu_\nu\) from \(O\) to \(O'\) in the matrix form:
\[
\begin{pmatrix}
1.25 & 0 & 0 & .75 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
.75 & 0 & 0 & 1.25
\end{pmatrix}
\]
(a) What is the velocity (speed and direction) of \(O'\) relative to \(O\)?
(b) What is the inverse matrix to the given one?
(c) Find the components in \(O\) of a vector \(A = (1, 2, 0, 0)\) given in the \(O'\) frame.

4. Let \(\Lambda^{\alpha}_{\beta}\) be the matrix of the Lorentz transformation parallel to x axis with a constant velocity \(v\) from \(O\) to \(O'\). Let \(A\) be an arbitrary vector with components \((A^0, A^1, A^2, A^3)\) in frame \(O\).
(a) Write down the matrix of \(\Lambda^{\nu}_{\bar{\mu}}\)
(b) Find \(A^{\bar{\alpha}}\) for all \(\bar{\alpha}\).
(c) Verify \(\Lambda^{\nu}_{\bar{\mu}}\Lambda^{\bar{\mu}}_{\beta} = \delta^{\nu}_{\beta}\).

5. Given \(A = (0, 2, 3, 5)\), find:
(a) the components of \(A\) in \(O'\), which moves at speed 0.6 relative to \(O\) in the positive x direction;
(b) the magnitude of \(A\) from its components in \(O\) and \(O'\).

6. (a) Compute the four-velocity components in \(O\) of a particle whose speed in \(O\) is \(v\) in the positive x direction, by using the Lorentz transformation from the rest frame of the particle.
(b) Generalize this result to find the four-velocity components when the particle has arbitrary velocity \(v\), with \(|v| < c\).
(c) Use your result in (b) to express $v$ in terms of the components $U^\alpha$. (d) Find the three-velocity $v$ of a particle whose four-velocity components are $(2, 1, 1, 1)$. 