Physics 218 – Exam III

Fall 2009, §517–520
(Melconian, T-Th 6:00–7:15 pm)

Name (printed): ___________________________ Section Number: __________

Signature: ________________________________

§517 Mon 8:00 10:50 am (Kechen Wang) §518 Mon 11:30 2:20 pm (Andrew Traverso)
§519 Mon 3:00 5:50 pm (Chi Chen) §520 Mon 6:10 9:00 pm (Jieyu Wang)

You may use any type of handheld calculator and should refer to the formula sheet which provides all the formulae/constants/conversions you will need for the problems in this exam.

This exam, marked out of 100%, consists of two parts:

1. four (4) short answer questions worth a total of 20%;
2. four (4) free-response problems worth a total of 80%; and
3. one (1) bonus (more difficult) free-response question worth a total of up to \((20 + 10) = 30\%\).

All question (short answer and free-response problems) require that your work be shown; you will not get credit for simply writing down the answer, whether it is correct or not.

We can’t give you part marks if we can’t understand what you’ve written! So please write legibly and present your work logically; it is in your best interest to make easy for us to understand what you did. If you need extra space, feel free to use the back of the last page, making sure you’ve clearly indicated that you have done so.
Short Answers:

1. [5 pts] A uniform rod of length $L$ and mass $M$ is pivoted at one end as shown. It is released from position $A$. Note that $\sin 36.9^\circ = \frac{4}{5}$ and $\cos 36.9^\circ = \frac{3}{5}$. Its angular acceleration at $B$ is closest to:
   
   (a) $\frac{9g}{40L}$  
   (b) $\frac{3g}{10L}$  
   (c) $\frac{3g}{5L}$  
   (d) $\frac{9g}{10L}$  
   (e) $\frac{6g}{5L}$  
   (f) $\frac{9g}{5L}$  

2. [5 pts] The ceiling fan shown is made up of five thin rectangular pieces of wood and a 0.50-kg stabilizing metal ring of radius $R = 0.40$ m. Each piece of wood has a mass 0.15 kg and the dimensions are $L = 0.80$ m by $w = 0.10$ m. The moment-of-inertia of this compound object around its centre and out of the page is closest to:
   
   (a) 0.083 Nm$^2$  
   (b) 0.112 Nm$^2$  
   (c) 0.121 Nm$^2$  
   (d) 0.200 Nm$^2$  
   (e) 0.240 Nm$^2$  
   (f) 0.274 Nm$^2$  

3. [5 pts] Last week while playing hockey, I was given a drop pass from a teammate in front of the net and scored with a one-timer slapshot (it was a beauty!). I estimate the 0.16 kg puck was sent to me at $v_0 = 2.0$ m/s in the negative $i$ direction and, after I hit it, the puck was going 18.0 m/s in the positive $i$ direction (we'll ignore the vertical component I gave it for simplicity). If my stick was in contact with the puck for 20 ms, what was the average force I hit it with?

4. [5 pts] A turntable made from a 2.40 kg solid disk is freely rotating around a frictionless pivot at 33.3 rpm as shown below. When I placed an initially motionless LP (which is a 0.18 kg solid disk of the same radius as the turntable) onto it, it slipped for 0.20 secs before catching and moving with the turntable. What is the final rotational speed of the system?
Problem 1  *Conservation of momentum* [20 pts]: A pebble moving at 15.0 m/s strikes a stationary rock that has five times the mass of the pebble. After the collision, the rock moves at 3.50 m/s at an angle of 11.3° with respect to the original line of motion.

(a) Find the velocity (magnitude and direction) of the pebble after the collision. Include a rough sketch to prove you know what the angle corresponds to.

Ans: ________________

(b) Was the collision elastic or inelastic? (Simply answering “elastic” or “inelastic” will not give you any marks; you have to back up your answer by showing your work).

Ans: ________________
Problem 2  Y8F 9.19 [20 pts]: At $t = 0$ a grinding wheel has an angular velocity of 21.0 rad/s. It has a constant angular acceleration of 27.0 rad/s$^2$ until a circuit breaker trips at $t = 1.90$ s. From then on, it turns through 405 radians as it coasts to a stop at constant angular acceleration.

(a) Through what total angle did the wheel turn between $t = 0$ and the time it stopped?

Ans: 

(b) At what time did it stop?

Ans: 

(c) What was its acceleration as it slowed down?

Ans: 
Problem 3 Rotation about a moving axis [20 pts]: A certain rock is very close to being a solid, uniform sphere with a diameter of 60 cm. It is sitting at a height 44 m at the top of a hill overlooking a valley as shown to the right.

(a) The terrain is rough enough to cause the rock to roll down the mountainside without slipping, but 3/4 of the way down it hits an oil slick which removes all friction for the rest of its entire motion. What is the translational speed of the rock when it reaches the valley at the bottom of the hill?

Ans: _______________________

(b) How high up the other side of the valley will the rock go?

Ans: _______________________

(c) Explain, using energy considerations, why your answer to (b) is not the 44 m height it started at.

Ans: _______________________
Problem 4  *Equilibrium of a rigid body* [20 pts]: A hungry 700-N Yogi Bear found a picnic basket filled with goodies hanging at the end of a 200-N uniform beam of length 7.00 m. The beam is angled and supported by a cable as shown in the figure to the right. “Hey Boo-Boo,” exclaimed Yogi as he proceeded to run towards his 90.0-N prize, “that pic-a-nic basket sure looks tasty!”

(a) When Yogi’s centre-of-mass is at \( x = 1.00 \) m along the beam, find the tension in the cable and the components of the force of the hinge on the beam.

(b) Even if Yogi is smarter than the average bear, he’s not smart enough ... the wire can only withstand 700 N before breaking. How far does Yogi get before the cable breaks?

**Ans:** 
________________________

________________________

________________________
Note: This question is considerably more difficult than the others, and part marks will be awarded much more sparingly.

Bonus [20 pts]: A point-like ball of mass $M_1 = 5.00$ kg is dropped from a height of $h = 10.6$ m above one end of a uniform bar that pivots at its centre. At the other end of the bar sits (unattached) another point-like ball with mass $M_2 = 4.80$ kg. The 4.40-m long bar has a mass of $M_{\text{bar}} = 7.50$ kg and is initially propped up on one end with an unattached stick so it is horizontal. The dropped ball sticks to the bar after the collision.

(a) Immediately following the collision, what is the angular velocity of the system?

(b) What maximum height, $h'$, will the 4.80-kg ball reach after the collision?

Ans: ________________________

Extra bonus [10 pts]: Derive the analytic expression for $h'$ in terms of the initial height and the three masses (use the back of this page to show your work).
Formule

General math:

\[
\log (x/y) = \log (x) - \log (y) \\
\log (xy) = \log (x) + \log (y) \\
\log (x^n) = n \log (x)
\]

\[
x = 10^{(\log_{10} x)} \\
x = e^{(\ln x)}
\]

\[
h_a = h \cos \theta = h \sin \phi \\
h_b = h \sin \theta = h \cos \phi \\
h^2 = h_a^2 + h_b^2 \\
\tan \theta = \frac{h_b}{h_a}
\]

\[
\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \\
\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = AB \cos \theta \\
\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k} \\
= AB \sin \theta = A_\perp B = B \perp
\]

If \( f(t) = at^n \), then

\[
\frac{df}{dt} = nat^{n-1} \\
f_t = \int_{t_1}^{t_2} f(t)dt = \frac{a}{n+1} \left( t_2^{n+1} - t_1^{n+1} \right)
\]

Translational Rotational

\[
\begin{align*}
\vec{r}(t) &= \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a}_0 t^2 \\
\vec{v}(t) &= \vec{v}_0 + \vec{a}_0 t \
\vec{a}(t) &= \frac{d\vec{a}}{dt} = \frac{d^2 \vec{r}}{dt^2} \\
\vec{v}^2 &= \vec{v}_0^2 + 2 \vec{a}_0 \cdot \vec{r} \
\vec{a}^2 &= \vec{a}_0^2 + 2 \vec{a}_0 \cdot \vec{v} \\
\vec{r}(t) &= \vec{r}_0 + \int_{t_0}^{t} \vec{v}(t) dt \\
\vec{v}(t) &= \vec{v}_0 + \int_{t_0}^{t} \vec{a}(t) dt \\
\vec{a}(t) &= \int_{t_0}^{t} \vec{\omega}(t) dt \\
\vec{\omega}(t) &= \theta(t) = \theta_0 + \alpha t + \frac{1}{2} \omega t^2
\end{align*}
\]

always true:

\[
\begin{align*}
\langle \vec{r} \rangle &= \frac{\vec{r}_0 + \vec{r}}{t_2 - t_1} \\
\langle \vec{v} \rangle &= \frac{\vec{v}_0 + \vec{v}}{t_2 - t_1} \\
\langle \vec{a} \rangle &= \frac{\vec{a}_0 + \vec{a}}{t_2 - t_1} \\
\langle \vec{\omega} \rangle &= \frac{\vec{\omega}_0 + \vec{\omega}}{t_2 - t_1} \\
\langle \alpha \rangle &= \frac{\alpha_0 + \alpha}{t_2 - t_1}
\end{align*}
\]

Circular motion:

\[
\begin{align*}
s &= R \theta \\
v_{\text{tan}} &= R \omega \\
o_{\text{tan}} &= R \alpha
\end{align*}
\]

\[
o_{\text{rad}} = \frac{\alpha}{R} \\
T = \frac{2\pi R}{\alpha}
\]

Relative velocity:

\[
\vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C}
\]

Constants/Conversions:

\[
g = 9.80 \text{ m/s}^2 = 32.15 \text{ ft/s}^2 \text{(on surface of Earth)}
\]

<table>
<thead>
<tr>
<th>Unit</th>
<th>Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 km</td>
<td>0.6214 mi</td>
</tr>
<tr>
<td>1 ft</td>
<td>0.3048 m</td>
</tr>
<tr>
<td>1 hr</td>
<td>3600 s</td>
</tr>
<tr>
<td>1 kg</td>
<td>2.248 lb</td>
</tr>
<tr>
<td>1 J</td>
<td>1 N·m</td>
</tr>
<tr>
<td>1 W</td>
<td>1 J/s</td>
</tr>
<tr>
<td>1 rev</td>
<td>360° = 2π rad</td>
</tr>
<tr>
<td>1 hp</td>
<td>745.7 W</td>
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<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Value</th>
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</thead>
<tbody>
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<td>nano-</td>
<td>n</td>
<td>10^-9</td>
</tr>
<tr>
<td>micro-</td>
<td>\mu</td>
<td>10^-6</td>
</tr>
<tr>
<td>milli-</td>
<td>m</td>
<td>10^-3</td>
</tr>
<tr>
<td>centi-</td>
<td>c</td>
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</tr>
<tr>
<td>kilo-</td>
<td>k</td>
<td>10^3</td>
</tr>
<tr>
<td>mega-</td>
<td>M</td>
<td>10^6</td>
</tr>
<tr>
<td>giga-</td>
<td>G</td>
<td>10^9</td>
</tr>
</tbody>
</table>

Newton’s Laws:

\[
\begin{align*}
\vec{F} &= m \vec{a} \\
\vec{F}_{\text{B on A}} &= -\vec{F}_{\text{A on B}} \\
\vec{w} &= -mg \hat{z} \\
\vec{F}_{\text{spring}} &= -k \Delta \vec{r} \\
|\vec{F}_\text{s}| &\leq \mu_s |\vec{F}| \\
|\vec{F}_\text{k}| &\leq \mu_k |\vec{F}|
\end{align*}
\]

Work-Energy:

\[
\begin{align*}
U_{\text{grav}} &= Mg y_{\text{cm}} \\
U_{\text{elas}} &= \frac{1}{2} k (x - x_{\text{equilibrium}})^2 \\
W_{\text{tot}} &= W_{\text{rot, i}} + W_{\text{other}} \\
W_{\text{rot}} &= \frac{1}{2} M \omega^2 \\
F_x &= -dU(x)/dx \\
\vec{F} &= -\nabla U = -\left[ \frac{\partial U}{\partial x} \hat{i} + \frac{\partial U}{\partial y} \hat{j} + \frac{\partial U}{\partial z} \hat{k} \right]
\end{align*}
\]
For a point-like particle of mass $M$ a distance $R$ from the axis of rotation, $I = MR^2$

Parallel axis theorem: $I_{p} = I_{cm} + Md^2$