Physics 218 – Final Exam

Fall 2012, §515–519, 562
(Melconian, TTh 5:30–6:45pm)

Name (printed): ___________________________  Section Number: _____________
Signature: __________________________________

You may use any type of handheld calculator and should refer to the sheet provided which lists all formulae/constants/conversions you will need for the problems in this exam (please return it if you don’t write on it and do not wish to keep it).

This exam consists of three parts:
1. Five (5) short-answer/multiple-choice questions worth a total of 15%;
2. Eight (8) free-response problems worth a total of 85%; and
3. Two (2) bonus questions worth a total of 10%.

Multiple-choice questions do not require any work be shown, and there is no penalty for guessing the wrong answer. However, all free-response and bonus problems require that your work be shown; you will not get credit for simply writing down the answer, whether it is correct or not.

We can’t give you part marks if we can’t understand what you’ve written! So please write legibly and present your work logically; it is in your best interest to make easy for us to understand what you did.

<table>
<thead>
<tr>
<th>Short answ</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>Bonus</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>9</td>
<td>12</td>
<td>10</td>
<td>12</td>
<td>12</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>100</td>
</tr>
</tbody>
</table>
Multiple Choice and Short Answers:

1. [3 pts] Is it possible for an object to be slowing down while its acceleration is increasing in magnitude? Briefly explain your reasoning.

2. [3 pts] The potential-energy function for a force is $U = \alpha x^3$, where $\alpha$ is a positive constant. What is the direction of the force?

3. [3 pts] A cylinder which has an inner diameter half the size of its outer diameter, a uniform solid cylinder and a thin-walled hoop are released from rest at the top of an incline. All objects roll without slipping. What is the order in which they arrive at the bottom of the incline?

4. [3 pts] A pendulum clock made by a company based at sea level in Amsterdam is bought by a Swiss tourist and taken to the top of a high mountain in the Alps. If the clock is made so that it tick-tocks the correct time where it is made, does still keep the correct time when brought to its new home? If so, explain why; if not, explain why it either gains or loses time.

5. [3 pts] Energy can be transferred along a string by wave motion. However, in a standing wave on a string, no energy can ever be transferred past a node. Why not?
Prob 1 [9 pts]: A 6.90-kg instrument is hanging by a vertical wire inside a space ship that is blasting off from the surface of the earth. This ship starts from rest and reaches an altitude of 290 m in 16.0 s with constant acceleration.
(a) Draw a free-body diagram for the instrument during this time. Make sure the relative lengths of your arrows reflect the relative strengths of the forces.

(b) Find the force that the wire exerts on the instrument.

Prob 2 Y8/F 3.56 (hmwk) [12 pts]: As a ship is approaching the dock at 45.0 cm/s, an important piece of landing equipment needs to be thrown to it before it can dock. This equipment is thrown at 12.0 m/s at 55.0° above the horizontal from the top of a tower at the edge of the water, a height $H = 9.50$ m above the ship’s deck. For this equipment to land at the front of the ship, at what distance $D$ from the dock should the ship be when the equipment is thrown? Air resistance may be neglected.

Ans:
Prob 3 [10 pts]: An 8.00-g bullet is shot into a 1.20-kg wooden block suspended on a string 2.00-m long. The bullet embeds in the block, and the combined object rises up following the completely inelastic collision. The string is found to make a maximum angle of 12.5° with respect to the vertical before swinging back down. Determine the initial speed of the bullet.

Ans:

Prob 4 [12 pts]: You are lowering two boxes, one on top of the other, down the ramp shown by pulling on a rope parallel to the surface of the ramp. Both boxes move together at a constant speed of 15.0 cm/s. The coefficient of kinetic friction between the ramp and the lower box is 0.444, and the coefficient of static friction between the two boxes is 0.800. The masses of the boxes are \( m_1 = 32.0 \) kg and \( m_2 = 48.0 \) kg, and the dimensions of the ramp are \( L = 4.75 \) m and \( H = 2.50 \) m.

(a) What force do you need to exert to accomplish this?

Ans:

(b) What are the magnitude and direction of the friction force on the upper box?

Ans:
Prob 5  Y&F 10.14 (hmwk) [12 pts] A stone is suspended from the free end of a wire that is wrapped around the outer rim of a pulley as shown. The impractical pulley can be approximated as a uniform sphere with mass $M = 12.00$ kg and radius $R = 50.0$ cm that turns on frictionless bearings. You measure that the stone travels $12.6$ m in the first $2.50$ s starting from rest.

(a) What is the mass of the stone?

(b) What is the tension in the wire as the stone descends?

Ans: ______________

Prob 6  Y&F Example 13.9 [10 pts]: Comet Halley moves in an elongated elliptical orbit around the sun. The distances of its centre-of-mass from the sun’s at perihelion and aphelion are $9.65 \times 10^7$ km and $2.96 \times 10^9$ km, respectively.

(a) What is the orbital semi-major axis of Comet Halley?

(b) What is the eccentricity of Halley’s orbit?

(c) Determine the period of the comet’s orbit.

Ans: ______________
Prob 7 [10 pts] A 40.0-N force stretches a vertical spring 0.225 m.
(a) What mass must be suspended from the spring so that the system will oscillate with a period of 1.00 s?

Ans: 

(b) If the amplitude of motion is 0.040 m and the period is 1.00 s, where is the object and in what direction is it moving 0.35 s after it has passed the equilibrium position and was moving downward?

Ans: 

Prob 8 YEF 15.41 (hmwk) [10 pts] A wire with mass 35.0 g is stretched so that its ends are tied down at points 120 cm apart. The wire vibrates in its fundamental mode with frequency 60.0 Hz and with an amplitude at the anti-nodes of 0.280 cm.
(a) What is the speed of propagation of transverse waves in the wire?

Ans: 

(b) Compute the tension in the wire

Ans: 

(c) Find the maximum transverse velocity and acceleration of particles in the wire

Ans: 
**Bonus Questions:** These question are not worth as much, the “value for work needed” is much worse, and there are no part marks...so you should only try them if you’re done early!

**Bonus 1** [5 pts]: Two children are playing a game in which they try to shoot a small marble of mass $m$ into a box a spring-loaded gun. They fix the gun rigidly to a table that is a height $h$ above the top of the box. The spring has a force constant $k$ and the centre of the box is some unknown horizontal distance $L$ away from where the marble leaves the spring. The first child compresses the spring a distance $x_0$ and finds that the marble falls short of its target by a horizontal distance $l$. How far should the 2nd child compress the spring in order to land in the box? Let $g$ denote the acceleration due to gravity. Express your answer in terms of $k$, $m$, $x$, $g$, $h$ and $l$ as needed; do not use the unknown distance $L$.

![Diagram of the game setup](image)

**Ans:**

**Bonus 2** [5 pts]: On Oct. 14 2012, Felix Baumgartner became the first person to break the sound barrier without any sort of engine; he “simply” parachuted from 39,045 m above the Earth and was in free-fall for 4 minutes and 19 seconds before he opened his parachute. The force of friction from high-speed air drag was $f_{drag} = Dv^2$, where you can assume (incorrectly) that $D$ was constant over his long drop and equal to $5.63 \times 10^{-3}$ kg/m. Also neglecting the small change in $g$ as he fell, derive an expression for his speed as a function of time and use it to calculate his maximum speed just before he opened his chute. Note that $\int \frac{dv}{v^2 - v_0^2} = \frac{1}{2a} \ln \left| \frac{v-a}{v+a} \right| + C$ and that the speed of sound, Mach 1, is 331.46 m/s.

![Diagram of Felix Baumgartner's jump](image)

**Ans:**
Phys 218 — Final Exam Formulae

General math:

\[ h_a = h \cos \theta = h \sin \phi \]
\[ h_o = h \sin \theta = h \cos \phi \]
\[ h^2 = h_o^2 + h_a^2 \quad \tan \theta = \frac{h_o}{h_a} \]
\[ \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \]
\[ \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = AB \cos \theta = A_1 B = AB \|
\[ \vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k} = AB \sin \theta = A_1 B = AB_\perp \]

If \( f(t) = at^n \), then
\[
\begin{cases}
\int_{t_1}^{t_2} f(t) dt = \frac{a}{n+1} (t_2^{n+1} - t_1^{n+1}) & (n \neq -1) \\
\int f(t) dt = \frac{a}{n+1} t^{n+1} + C & (n \neq -1)
\end{cases}
\]

Equations of motion:

**translational**

— constant (linear/angular) acceleration only —
\[
\begin{align*}
\vec{r}(t) &= \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 \\
\vec{v}(t) &= \vec{v}_0 + \vec{a} t \\
v^2 \vec{r} &= v^2_{r,0} + 2ax(x-x_0) \\
\vec{r}(t) &= \vec{r}_0 + \frac{1}{2} (\vec{v}_0 + \vec{v}(t)) t
\end{align*}
\]
\( \vec{v} \) always true
\( \vec{a} \) always true
\[ \vec{a} = \frac{d^2 \vec{r}}{dt^2} = \frac{d^2 \vec{v}}{dt^2} \quad \vec{a} = \frac{d^2 \vec{r}}{dt^2} = \frac{d^2 \vec{v}}{dt^2} \]
\[ \vec{r}(t) = \vec{r}_0 + \int_0^t \vec{v}(t') dt' \quad \vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a}(t') dt' \]

**rotational**

\[ \vec{\tau} = \vec{r} \times \vec{F} \quad \text{and} \quad |\vec{\tau}| = F L \]
\[ K_{\text{trans}} = \frac{1}{2} M v^2 \quad K_{\text{rot}} = \frac{1}{2} I \omega \]
\[ W = \int \vec{F} \cdot \vec{dr} \quad \text{const force} \]
\[ P = \frac{dW}{dt} = \vec{F} \cdot \vec{\omega} \]
\[ \vec{p}_{\text{cm}} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \ldots \]
\[ \vec{J} = \int \vec{F} dt = \Delta \vec{p} \]
\[ \sum \vec{F}_{\text{ext}} = M \vec{a}_{\text{cm}} = \frac{d\vec{p}_{\text{cm}}}{dt} \]
\[ \sum \vec{F}_{\text{int}} = 0 \quad \sum \vec{\tau}_{\text{int}} = 0 \]

—— Both translational and rotational ——
\[ W = \Delta K = K_{\text{trans},f} + K_{\text{rot},f} - K_{\text{trans},i} - K_{\text{rot},i} \]
\[ E_{\text{tot},f} = E_{\text{tot},i} + W_{\text{other}} \quad K_f + U_f = K_i + U_i + W_{\text{other}} \]

\[ U = -\int \vec{F} \cdot d\vec{r} \quad U_{\text{grav}} = Mg y_{\text{cm}} \quad U_{\text{class}} = \frac{1}{2} k (r - r_{\text{equi}})^2 \]
\[ F_x(x) = -dU(x)/dx \quad \vec{F} = -\vec{\nabla} U = -\left( \frac{\partial U}{\partial x} \hat{i} + \frac{\partial U}{\partial y} \hat{j} + \frac{\partial U}{\partial z} \hat{k} \right) \]

Circular motion:

\[ |\vec{a}_{\text{rad}}| = \frac{v^2}{R} \quad T = \frac{2\pi R}{v} \]
\[ s = R\theta \quad v_{\text{tan}} = R\omega \quad a_{\text{tan}} = R\alpha \]

Relative velocity:

\[ \vec{v}_{A/C} = \vec{v}_{A/B} + \vec{v}_{B/C} \]
\[ \vec{v}_{A/B} = -\vec{v}_{B/A} \]

Constants/Conversions:

\[ g = 9.80 \text{ m/s}^2 = 32.15 \text{ ft/s}^2 \text{ (on Earth’s surface)} \]
\[ G = 6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \]
\[ R_{\oplus} = 6.38 \times 10^6 \text{ m} \quad M_{\oplus} = 5.98 \times 10^{24} \text{ kg} \]
\[ R_{\odot} = 6.96 \times 10^8 \text{ m} \quad M_{\odot} = 1.99 \times 10^{30} \text{ kg} \]
\[ 1 \text{ km} = 0.6214 \text{ mi} \quad 1 \text{ mi} = 1.609 \text{ km} \]
\[ 1 \text{ ft} = 0.3048 \text{ m} \quad 1 \text{ m} = 3.281 \text{ ft} \]
\[ 1 \text{ hr} = 3600 \text{ s} \quad 1 \text{ s} = 0.0002778 \text{ hr} \]
\[ 1 \text{ kg m}^2/s^2 = 1 \text{ N} \quad 1 \text{ lb} = 4.448 \text{ N} \]
\[ 1 \text{ J} = 1 \text{ N} \cdot \text{m} \quad 1 \text{ W} = 1 \text{ J/s} \]
\[ 1 \text{ rev} = 360^\circ = 2\pi \text{ radians} \quad 1 \text{ hp} = 745.7 \text{ W} \]

Forces: Newton’s:
\[ \sum \vec{F} = m \vec{a} \quad \vec{F}_B \text{ on } A = -\vec{F}_A \text{ on } B \]

Hooke’s:
\[ \vec{F}_{\text{class}} = -k (r - r_{\text{equi}}) \vec{r} \quad \text{friction:} \quad |\vec{f}_s| \leq \mu_s |\vec{r}|, \quad |\vec{f}_k| = \mu_k |\vec{n}| \]

Centre-of-mass:

\[ \vec{r}_{\text{cm}} = m_1 \vec{r}_1 + m_2 \vec{r}_2 + \ldots + m_n \vec{r}_n \]
\[ m_1 + m_2 + \ldots + m_n \]

—and similarly for \( \vec{v} \) and \( \vec{a} \)

Gravity:

\[ \vec{F}_{\text{grav}} = -G \frac{M_1 M_2}{r^{12}} \hat{r} \quad U_{\text{grav}} = -G \frac{M_1 M_2}{R_{12}} \quad T = \frac{2\pi a^{3/2}}{\sqrt{GM}} \]

Perihelion

\[ S \quad P \quad \text{Aphelion} \]

<table>
<thead>
<tr>
<th>S</th>
<th>O</th>
<th>P</th>
<th>S'</th>
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</thead>
</table>

a | a | ea | ea |
### Table 9.2Moments of Inertia of Various Bodies

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I = \frac{1}{12}ML^2 )</td>
<td>slender rod, axis through centre</td>
</tr>
<tr>
<td>( I = \frac{1}{3}ML^2 )</td>
<td>slender rod, axis through one end</td>
</tr>
<tr>
<td>( I = \frac{1}{12}M(a^2+b^2) )</td>
<td>rectangular plate, axis through centre</td>
</tr>
<tr>
<td>( I = \frac{1}{3}Ma^2 )</td>
<td>thin rectangular plate, axis along an edge</td>
</tr>
<tr>
<td>( I = \frac{1}{2}M(R_1^2+R_2^2) )</td>
<td>hollow cylinder</td>
</tr>
<tr>
<td>( I = \frac{1}{2}MR^2 )</td>
<td>solid cylinder</td>
</tr>
<tr>
<td>( I = MR^2 )</td>
<td>thin-walled hollow cylinder</td>
</tr>
<tr>
<td>( I = \frac{2}{5}MR^2 )</td>
<td>solid sphere</td>
</tr>
<tr>
<td>( I = \frac{2}{3}MR^2 )</td>
<td>thin-walled hollow sphere</td>
</tr>
</tbody>
</table>

\(~\sim\) For a point-like particle of mass \( M \) a distance \( R \) from the axis of rotation: \( I = MR^2 \)

\(~\sim\) Parallel axis theorem: \( I_p = I_{cm} + Md^2 \)

<table>
<thead>
<tr>
<th>( \omega = 2\pi f = \frac{2\pi}{T} )</th>
<th>( \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{\text{pend}} = 2\pi\sqrt{\frac{L}{g}} = \frac{2\pi}{\sqrt{I/mgd}} )</td>
<td>( A = \frac{F_{\text{max}}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^4}} )</td>
</tr>
<tr>
<td>( T_{\text{spring}} = 2\pi\sqrt{\frac{m}{k}} )</td>
<td>( x(t) = A \cos (\omega't)e^{-(b/2m)t} )</td>
</tr>
<tr>
<td>( T_{\text{torsion}} = 2\pi\sqrt{\frac{I}{\kappa}} )</td>
<td>( x(t) = A \cos (\omega t + \phi) )</td>
</tr>
<tr>
<td>( v(t) = -\omega A \sin (\omega t + \phi) )</td>
<td>( v(t) = -\omega' A \sin (\omega't) )</td>
</tr>
<tr>
<td>( a(t) = -\omega^2 A \cos (\omega t + \phi) )</td>
<td>( a(t) = -\omega'^2 A \cos (\omega't) )</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Waves:</th>
<th>Damped/forced:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k = 2\pi/\lambda )</td>
<td>( \lambda_n = \frac{2L}{n} \leftrightarrow f_n = n\frac{v}{2L}, n = 1, 2, 3, \ldots )</td>
</tr>
<tr>
<td>( v = \pm \lambda f )</td>
<td>( n = 1, 2, 3, \ldots )</td>
</tr>
<tr>
<td>( v_{\text{string}} = F_T/\mu )</td>
<td>( \mu = M/L )</td>
</tr>
<tr>
<td>( \mu = M/L )</td>
<td>( \omega = \sqrt{\frac{k}{m}} )</td>
</tr>
</tbody>
</table>

Standing wave:

| \( y(x, t) = A_{SW} \sin (kx) \sin (\omega t) \) | \( P = \sqrt{\mu F_T \omega^2 A^2 \sin^2 (kx + \omega t)} \) |

Travelling wave:

| \( x(t) = A \cos \left[ 2\pi \left( \frac{x}{\lambda} + \frac{t}{T} \right) \right] \) | \( I(r) = \frac{\text{power}}{\text{surf area at } r} \left( = \frac{P}{4\pi r^2} \text{ for 3D sphere} \right) \) |

\( x_n(t) = A' \cos \left( \omega' t + \phi_n \right) \)